# **STRAND E: Measurement**

# Unit 13 Areas

# **Student Text**

## Contents

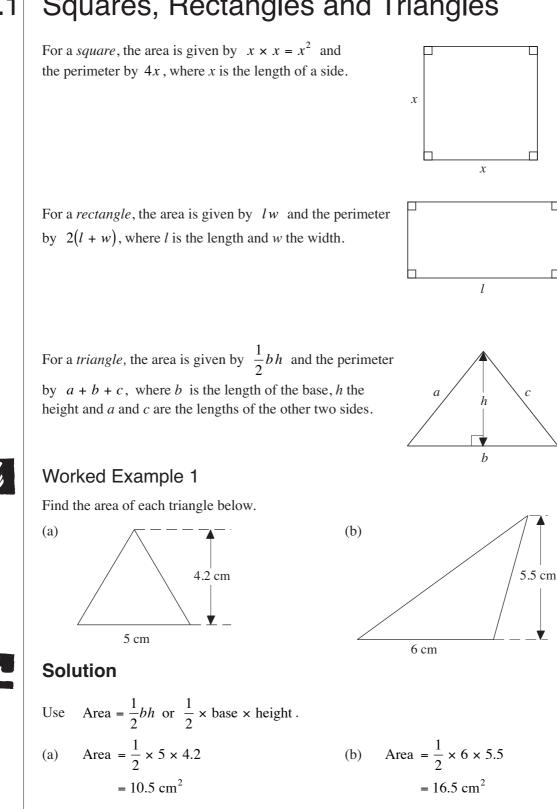
#### Section

13.1	Squares, Rectangles and Triangles
13.2	Area and Circumference of Circles
13.3	Sector Areas and Arc Lengths
13.4	Areas of Parallelograms, Trapeziums, Kites and Rhombuses
13.5	Surface Area

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# 13 **Areas**

#### 13.1 Squares, Rectangles and Triangles

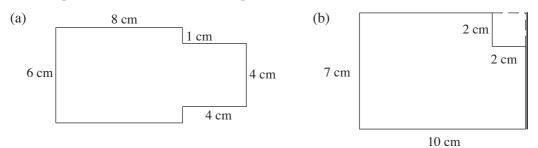




# (i)

# Worked Example 2

Find the perimeter and area of each shape below.





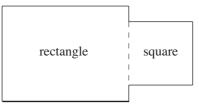
### Solution

(a) The perimeter is found by adding the lengths of all the sides.

$$P = 6 + 8 + 1 + 4 + 4 + 4 + 1 + 8$$

= 36 cm

To find the area, consider the shape split into a rectangle and a square.



Area = Area of rectangle + Area of square

$$= 6 \times 8 + 4^{2}$$
  
= 48 + 16  
= 64 cm<sup>2</sup>

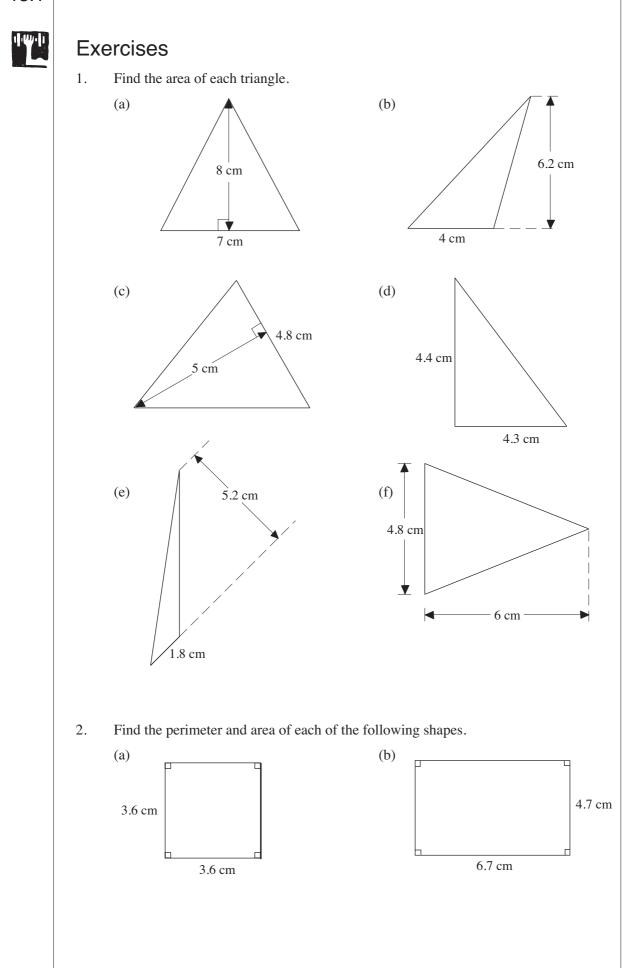
(b) Adding the lengths of the sides gives

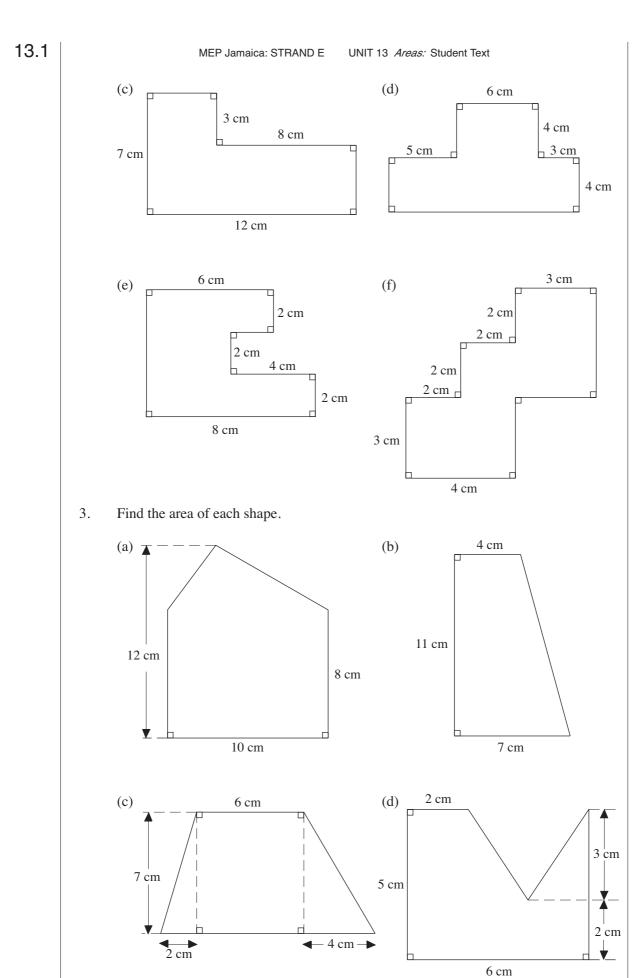
$$P = 10 + 7 + 8 + 2 + 2 + 5$$
$$= 34 \text{ cm}$$

The area can be found by considering the shape to be a rectangle with a square removed from it.

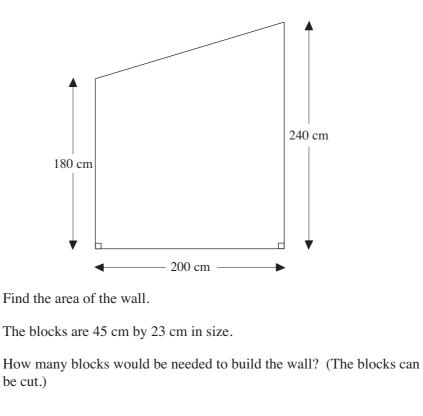
Area of shape = Area of rectangle - Area of square

= 
$$7 \times 10 - 2^2$$
  
=  $70 - 4$   
=  $66 \text{ cm}^2$ 

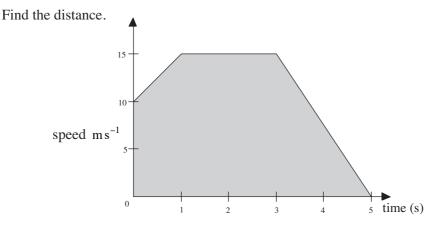




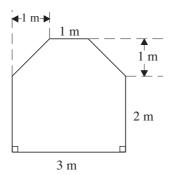
4. The diagram shows the end wall of a shed built out of concrete bricks.



5. The shaded area on the speed time graph represents the distance travelled by a bicycle.



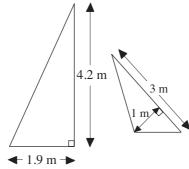
The plan shows the base of a storeroom.
 Find the area of the base.



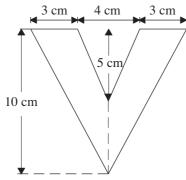
(a)

(b)

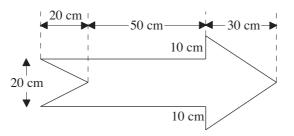
 The diagram shows the two sails from a small sailing boat. Find their combined area.



8. The diagram shows the letter V. Find the area of this letter.



9. Find the area of the arrow shown in the diagram.

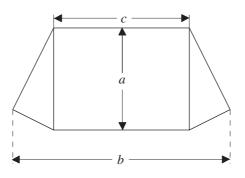


10. The diagram shows how the material required for one side of a tent is cut out.

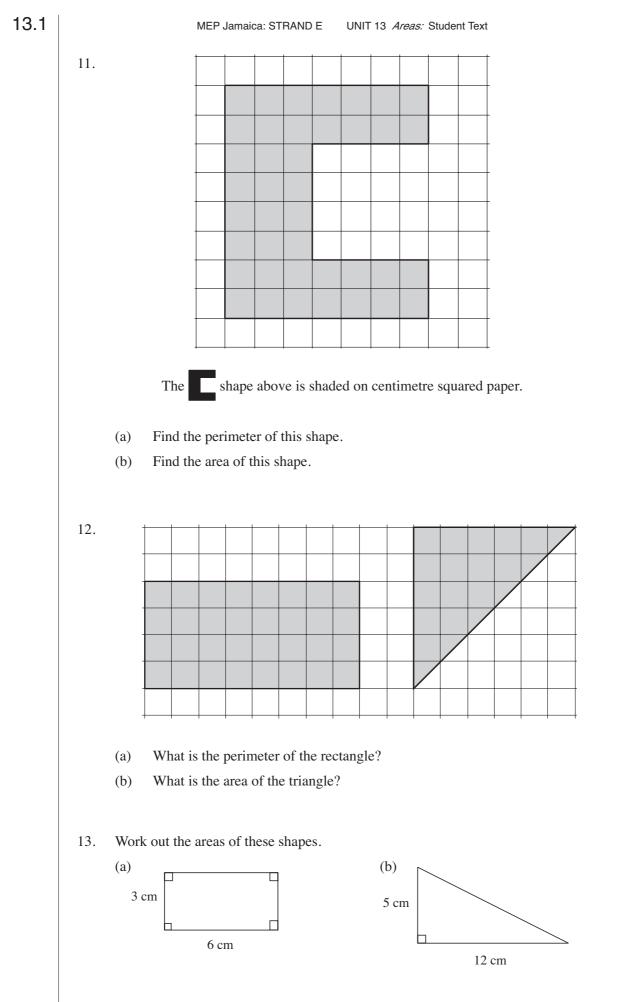
(a) Find the area of the material shown if b = 3.2 m, c = 2 m and

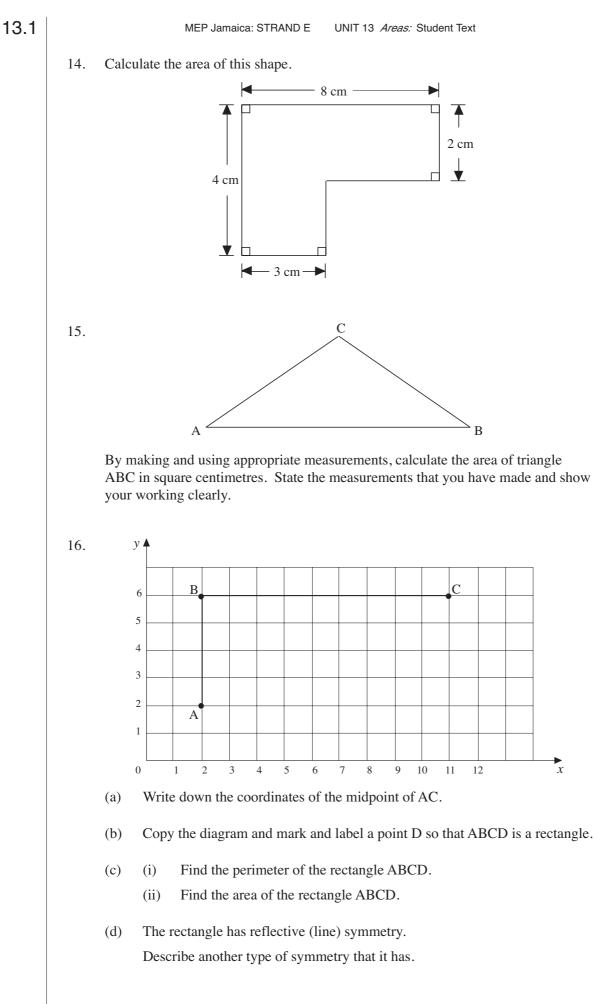
$$a = 1.5 \text{ m}$$
 (ii)  $a = 2 \text{ m}$ 

(b) Find the area if a = 1.6 m, b = 3.4 m and c = 2 m.



(i)





#### 13.2 Area and Circumference of Circles

The circumference of a circle can be calculated using

 $C = 2\pi r$  or  $C = \pi d$ 

where *r* is the radius and *d* the diameter of the circle.

The area of a circle is found using

$$A = \pi r^2$$
 or  $A = \frac{\pi d^2}{4}$ 



## Worked Example 1

Find the circumference and area of this circle.

### Solution

The circumference is found using  $C = 2\pi r$ , which in this case gives

 $C = 2\pi \times 4$ 

= 25.1 cm(to one decimal place)

The area is found using  $A = \pi r^2$ , which gives

 $A = \pi \times 4^2$ 

$$\sim$$
 +

 $= 50.3 \text{ cm}^2$  (to one decimal place)

# Worked Example 2

Find the radius of a circle if:

its circumference is 32 cm, (a)

```
(b) its area is 14.3 \text{ cm}^2.
```

# Solution

Using  $C = 2\pi r$  gives (a)

$$32 = 2\pi r$$

and dividing by  $2\pi$  gives

$$\frac{32}{2\pi} = r$$

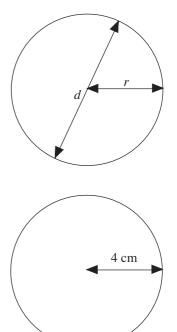
so that

r = 5.09 cm (to 2 decimal places)

(b) Using 
$$A = \pi r^2$$
 gives

 $14.3 = \pi r^2$ 





Dividing by  $\pi$  gives

$$\frac{14.3}{\pi} = r^2$$

Then taking the square root of both sides gives

$$\sqrt{\frac{14.3}{\pi}} = r$$

so that

$$r = 2.13$$
 cm (to 2 decimal places)



# Worked Example 3

Find the area of the door shown in the diagram. The top part of the door is a semicircle.



### Solution

First find the area of the rectangle.

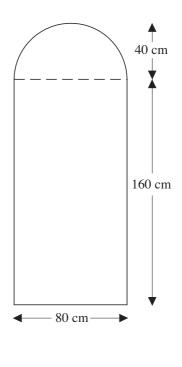
Area =  $80 \times 160$ =  $12800 \text{ cm}^2$ 

= 12800 cm

Then find the area of the semicircle.

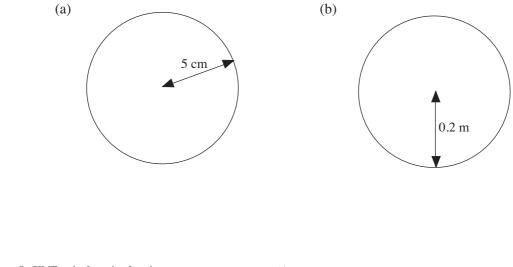
Area = 
$$\frac{1}{2} \times \pi \times 40^2$$
  
= 2513 cm<sup>2</sup>

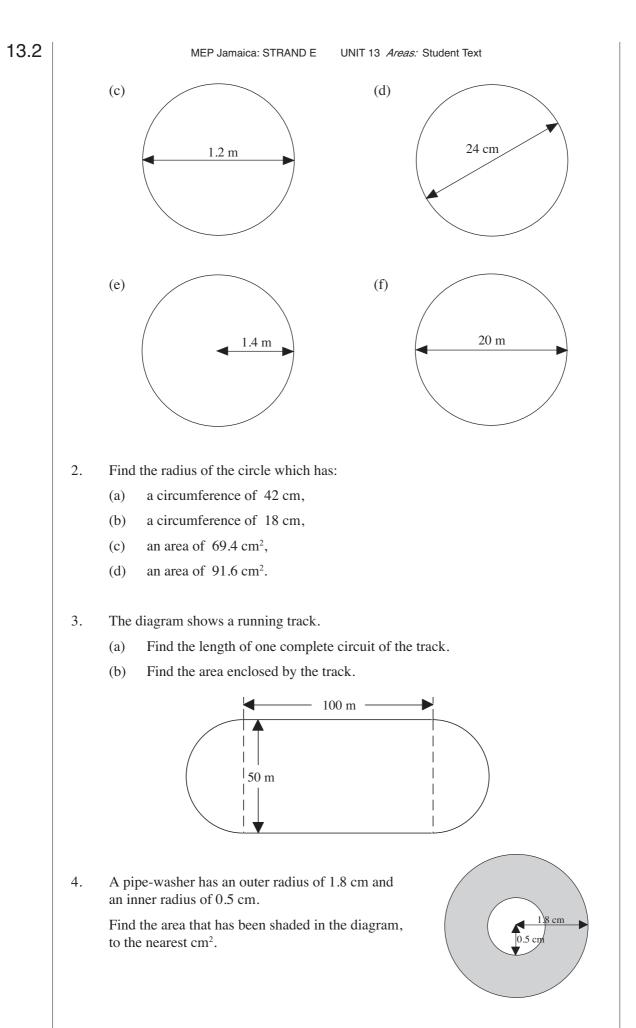
Total area = 12800 + 2513=  $15313 \text{ cm}^2$  (to the nearest cm<sup>2</sup>)



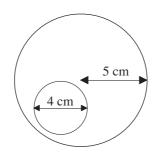
## **Exercises**

1. Find the circumference and area of each of the following circles.





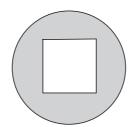
5. An egg, fried perfectly, can be thought of as a circle (the yolk) within a larger circle (the white).



- (a) Find the area of the smaller circle that represents the surface of the yolk.
- (b) Find the area of the surface of the whole egg.
- (c) Find the area of the surface of the white of the egg, to the nearest  $cm^2$ .
- The shapes shown below were cut out of card, ready to make cones.
   Find the area of each shape.



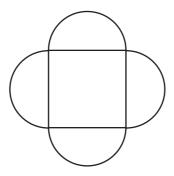
- 7. A circular hole with diameter 5 cm is cut out of a rectangular metal plate of length 10 cm and width 7 cm. Find the area of the plate when the hole has been cut out.
- 8. Find the area of the wasted material if two circles of radius 4 cm are cut out of a rectangular sheet of material that is 16 cm long and 8 cm wide.
- 9. A square hole is cut in a circular piece of card to create the shape shown.



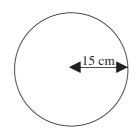
- (a) Find the shaded area of the card if the radius of the circle is 5.2 cm and the sides of the square are 4.8 cm.
- (b) Find the radius of the circle if the shaded area is 50 cm<sup>2</sup> and the square has sides of length 4.2 cm.

13.2

10. Four semicircles are fixed to the sides of a square as shown in the diagram, to form a design for a table top.



- (a) Find the area of the table top if the square has sides of length 1.5 m.
- (b) Find the length of the sides of the square and the total area of the table top if the area of each semicircle is  $1 \text{ m}^2$ .
- 11. The radius of a circle is 8 cm.
  Work out the area of the circle.
  (Use π = 3.14 or the π button on your calculator.)
- 12. A circle has a radius of 15 cm.



Calculate the area of the circle.

Take  $\pi$  to be 3.14 or use the  $\pi$  key on your calculator.

13. Lecia does a sponsored bicycle ride.

Each wheel of her bicycle is of radius 25 cm.

- (a) Calculate the circumference of one of the wheels
- (b) She cycles 50 km. How many revolutions does a wheel make during the sponsored ride?



14. The diameter of a garden roller is 0.4 m.

13.2

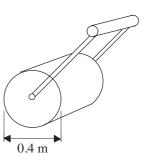
15.

16.

The roller is used on a path of length 20 m.

Calculate how many times the roller rotates when rolling the length of the path once.

Take  $\pi$  to be 3.14 or use the  $\pi$  key on your



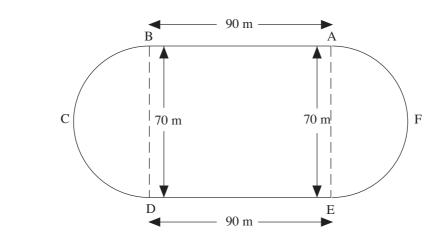
calculator.

A piece of rope is 12 metres long. It is laid on the ground in a circle, as shown in the diagram.

- Using 3.14 as the value of  $\pi$ , (a) calculate the diameter of the circle.
- (b) Explain briefly how you would check the answer to part (a) mentally.

The cross-section of the rope is a circle of radius 1.2 cm.

(c) Calculate the area of the cross-section.



The diagram shows a running track.

BA and DE are parallel and straight. They are each of length 90 metres.

BCD and EFA are semicircular. They each have a diameter of length 70 metres.

Using 
$$\pi = \frac{22}{7}$$
, calculate

(a) the perimeter of the track,

(b) the total area inside the track.

# 13.3 Sector Areas and Arc Lengths

A part of the circumference of a circle is called an *arc*. If the angle subtended by the arc at the centre of the circle is  $\theta$  then the arc length *l* is given by

$$l = \frac{\theta}{360^{\circ}} \times 2\pi r$$

The region between the two radii and the arc is called a *sector* of the circle. The area of the sector of the circle is

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$



# Worked Example 1

The shaded area shows a segment of a circle of radius 64 cm.

The length of the chord AB is 100 cm.

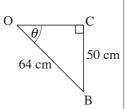
- (a) Find the angle  $\theta$ , to 2 d.p.
- (b) Find the area of triangle OAB.
- (c) Find the area of the sector of the circle with angle  $2\theta$ .
- (d) Find the area of the segment shaded in the figure.

### **Solution**

(a) If AB = 100 cm then, by symmetry, BC = 50 cm.

$$\sin\theta = \frac{50}{64}$$
$$\theta = 51.38^{\circ}$$

(b) The area of the triangle OAB is  $\frac{1}{2} \times 64^2 \times \sin 2\theta = 1997 \text{ cm}^2$ (This is using the result area of triangle  $= \frac{1}{2}ab\sin\theta$ , which is covered in Unit 34.)



arc

sector

В

Р

O

 $O \left< \frac{\theta}{\theta} \right> O$ 

64 cm

(c) The sector has area 
$$\frac{(2 \times 51.38^{\circ})}{360^{\circ}} \times \pi \times 64^2 = 3673 \text{ cm}^2$$

(d) The segment has area  $3673 - 1997 = 1676 \text{ cm}^2$ .



# 1 in

# Worked Example 2

A wooden door wedge is in the shape of a sector of a circle of radius 10 cm with angle  $24^{\circ}$  and constant thickness 3 cm.

Find the volume of wood used in making the wedge.\*

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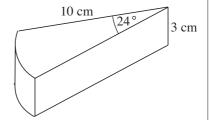
# Solution

The area of the top face of the wedge is the area of a sector of radius 10 cm and angle  $24^{\circ}$ .

Area = 
$$\frac{24^{\circ}}{360^{\circ}} \times \pi \times 10^2 = \frac{20\pi}{3} = 20.94 \text{ cm}^2$$

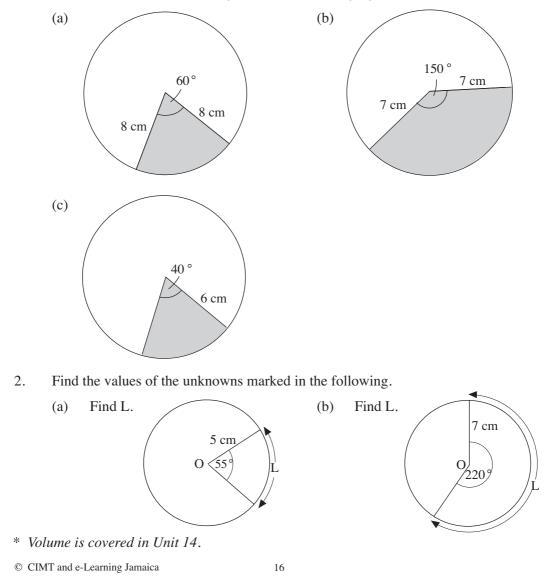
The volume of the wedge

= Area  $\times$  3 = 20 $\pi$  = 62.83 cm<sup>3</sup>



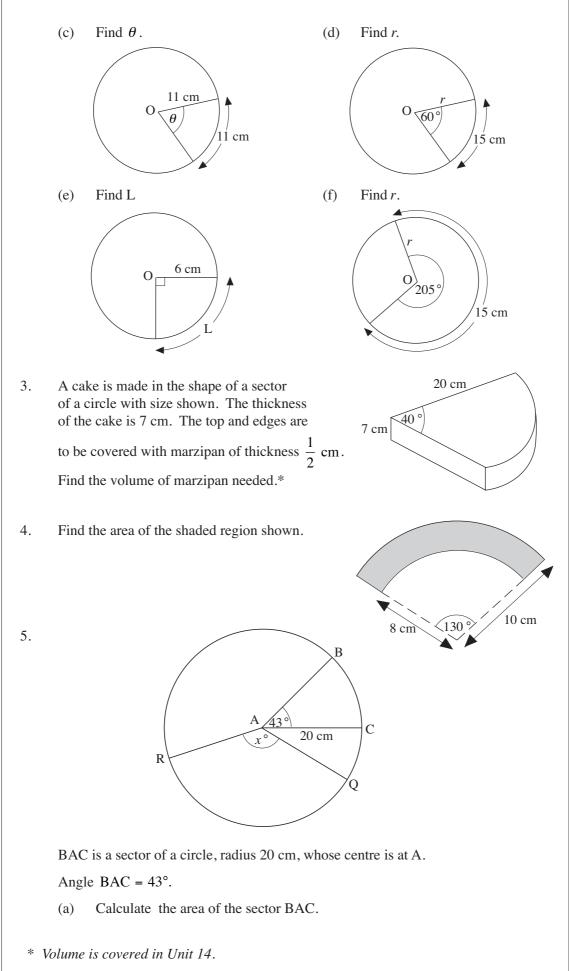
# Exercises

1. Find the area of the shaded regions in the following figures.



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13.3
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(b) The area of sector QAR is  $450 \text{ cm}^2$ .

Angle QAR is  $x^{\circ}$ .

13.3

Calculate the value of *x*.

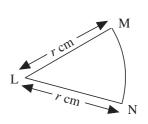
(c) The area of the sector MLN of another circle, centre L, is  $600 \text{ cm}^2$ .

The total perimeter of the sector is 100 cm.

It can be shown that the radius, r cm, of the sector satisfies the equation

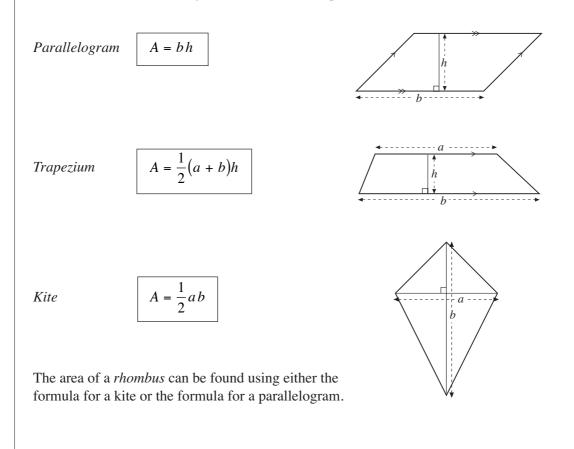
 $r^2 - 50r + 600 = 0$ 

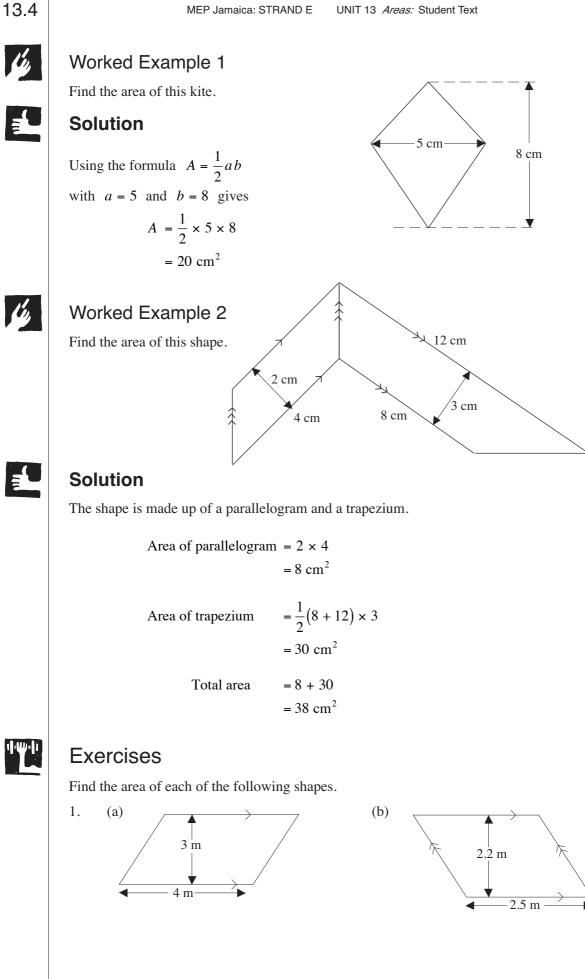
Find the values of *r* which satisfy this equation.

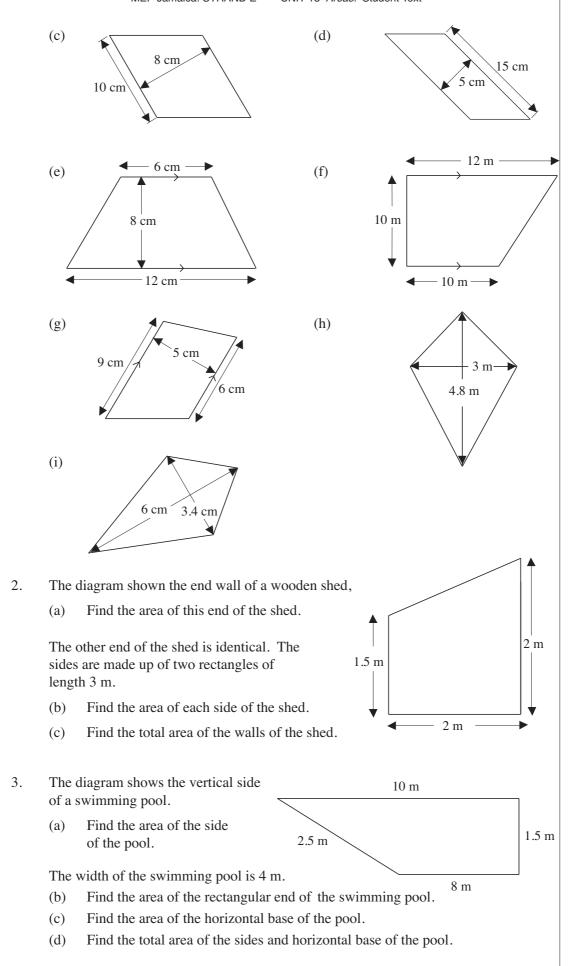


# 13.4 Areas of Parallelograms, Trapeziums, Kites and Rhombuses

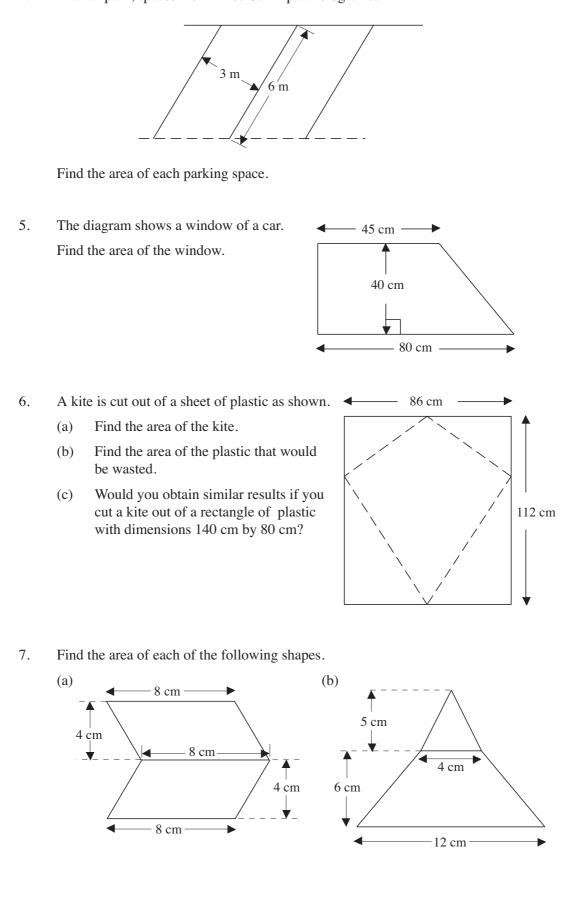
The formulae for calculating the areas of these shapes are:

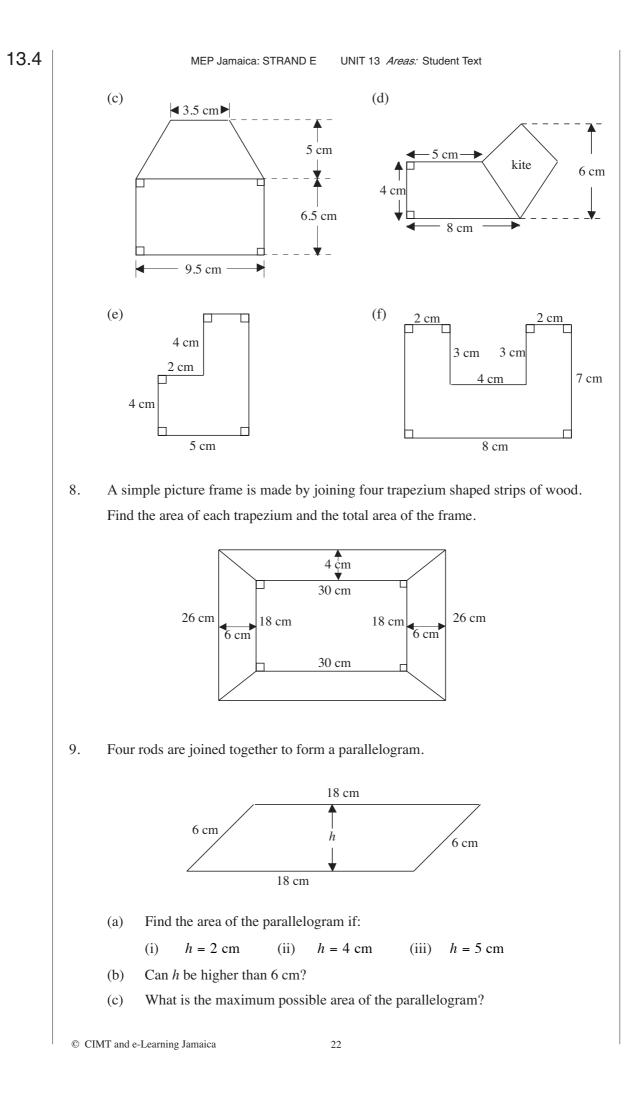


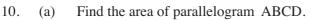




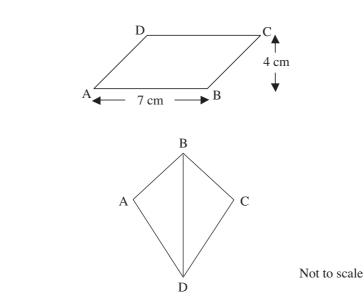
4. In a car park, spaces are marked out in parallelograms.







(b) Find the area of the triangle ABC.

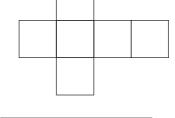


Why is the area of the kite ABCD equal to twice the area of the triangle ABD?

# 13.5 Surface Area

The *net* of a cube can be used to find its surface area.

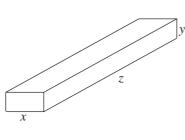
The net is made up of 6 squares, so the surface area will be 6 times the area of one square. If x is the length of the sides of the cube its surface area will be  $6x^2$ .

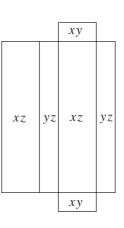


This diagram shows the net for a cuboid. To find the surface area the area of each of the 6 rectangles must be found and then added to give the total.



If *x*, *y* and *z* are the lengths of the sides of the cuboid, then the area of the rectangles in the net are as shown here.



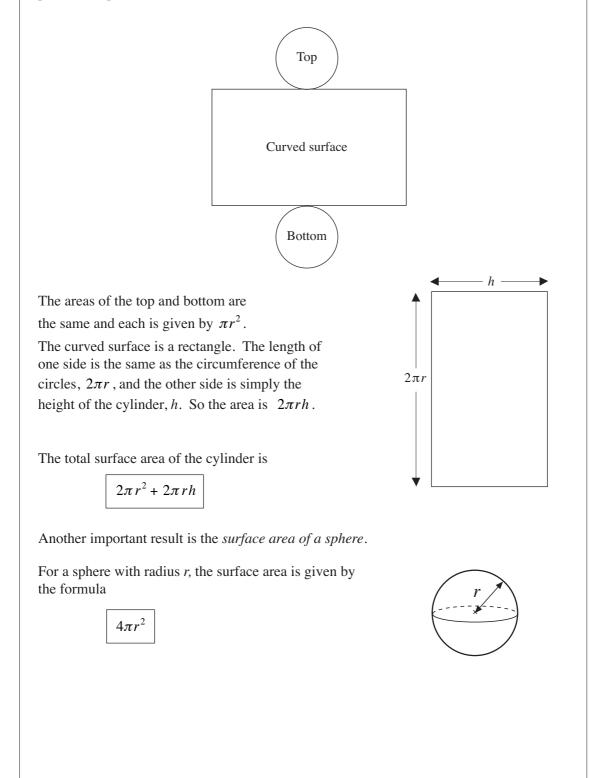


11.

The total surface area of the cuboid is then given by

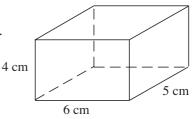
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A = 2xy + 2xz + 2yz
```

To find the surface area of a cylinder, consider how a cylinder can be broken up into three parts, the top, bottom and curved surface.



# Worked Example 1

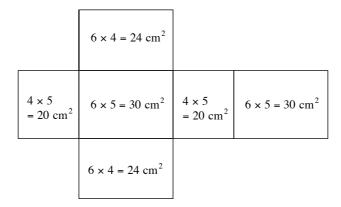
Find the surface area of the cuboid shown in the diagram.



# Solution

13.5

The diagram shows the net of the cuboid and the areas of the rectangles that it contains.



Using the net, the total surface area is given by

$$A = 2 \times 20 + 2 \times 30 + 2 \times 24$$
  
= 148 cm<sup>2</sup>

# Worked Example 2

Cans are made out of aluminium sheets, and are cylinders of radius 3 cm and height 10 cm. Find the area of aluminium needed to make one can.

# So

# Solution

The diagram shows the two circles and the rectangle from which cans will be made.

The rectangle has one side as 10 cm, the height of the cylinder and the other side is  $2 \times \pi \times 3$  cm, the circumference of the top and bottom.

be circles and the will be made. as 10 cm, the the other side is derence of the  $10 \times 2 \times \pi \times 3$  $\pi \times 3^2$  $A = 10 \times 2 \times \pi \times 3 + 2 \times \pi \times 3^2$ 

3 cm

The area of the rectangle is  $10 \times 2 \times \pi \times 3$ The area of each circle is  $\pi \times 3^2$ 

So the total surface area is

 $= 245.04 \text{ cm}^2$  (to 2 d.p.)



# Worked Example 3

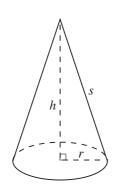
A ball has radius 4 cm. What is its surface area, to the nearest  $cm^2$ ?

### Solution

Surface area =  $4\pi r^2 \text{ cm}^2$  $= 4\pi 4^2 \text{ cm}^2$  $64\pi$  cm<sup>2</sup>  $201 \text{ cm}^2$  to the nearest  $\text{cm}^2$ 

#### Note

There is a formula for calculating the surface area of a **cone**:



surface area of cone =  $\pi rs + \pi r^2$ 

#### where

s = slant height of the cone

r = radius of the base

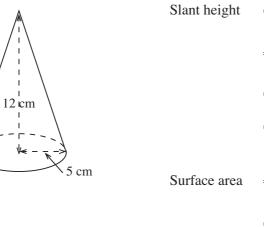
(and  $s^2 = h^2 + r^2$ , where *h* is the perpendicular height of the cone).



# Worked Example 4

What is the surface area of a cone of base radius 5 cm and perpendicular height 12 cm? Give your answer in terms of  $\pi$ .

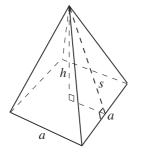
# Solution



at height = 
$$\sqrt{12^2 + 5^2}$$
  
=  $\sqrt{144 + 25}$   
=  $\sqrt{169}$   
= 13 cm  
Face area =  $(\pi \times 5 \times 13 + \pi \times 5^2)$  cm<sup>2</sup>  
=  $(65\pi + 25\pi)$  cm<sup>2</sup>  
=  $90\pi$  cm<sup>2</sup>

=

There is also a formula for calculating the surface area of a square-based pyramid:



where

s = perpendicular slant height of the pyramid
a = length of the side of the square base
(and h is the perpendicular height of the pyramid).

surface area =  $2as + a^2$ 

Note that  $a^2$  is the surface area of the base and each trianglar face has area  $\frac{1}{2}as$ .



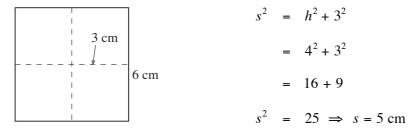
## Worked Example 5

What is the surface area of a square-based pyramid of base side 6 cm and height 4 cm?



### Solution

We first calculate the slant height from



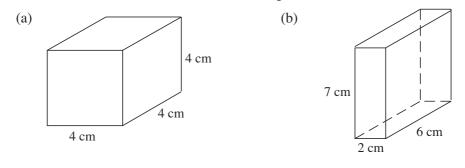
Hence,

surface area = 
$$(2 \times 6 \times 5 + 6^2) \text{ cm}^2$$
  
=  $(60 + 36) \text{ cm}^2$   
=  $96 \text{ cm}^2$ 



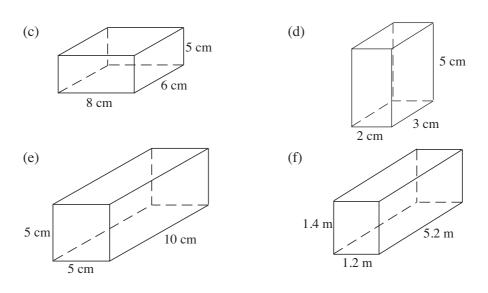
# Exercises

1. Find the surface area of each of the following cubes or cuboids.

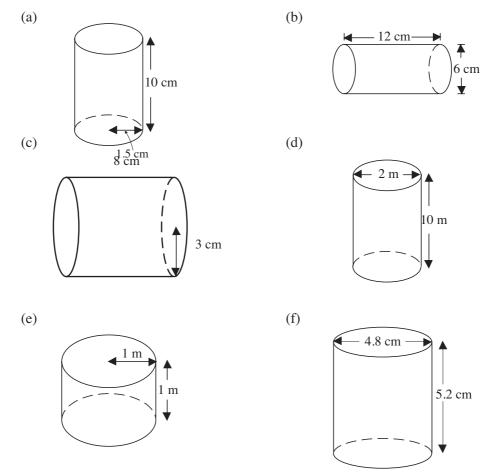


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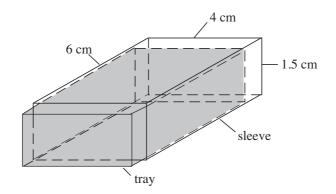
2. Find the total surface area of each cylinder shown below.



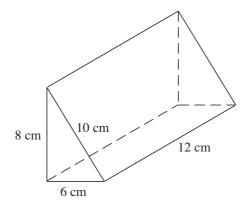
- 3. A groundsman uses a roller to compact the surface of a cricket pitch. The roller consists of a cylinder of radius 30 cm and width 70 cm.
  - (a) Find the area of ground that the roller covers as the cylinder completes 1 rotation.
  - (b) If the roller is pulled 5 m, what area of ground does the roller cover?

- 4. A matchbox consists of a tray that slides into a sleeve. If the tray and sleeve have the same dimensions and no material is used up in joins, find:
  - (a) the area of cardboard needed to make the tray,

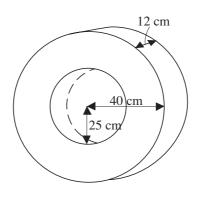
- (b) the area of cardboard needed to make the sleeve,
- (c) the total area of the cardboard needed to make the matchbox.

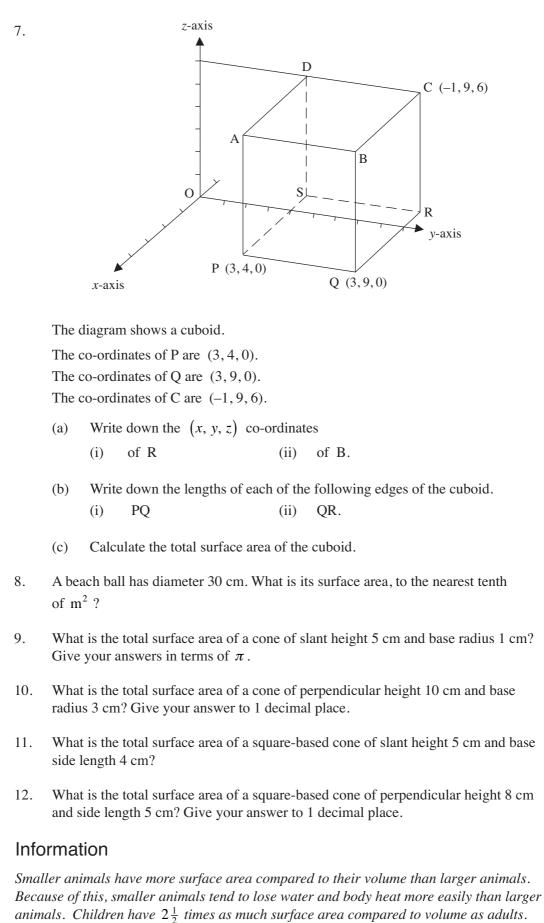


5. Draw a net of the prism shown in the diagram below and use it to find the surface area of the prism.



6. A car tyre can be thought of as a hollow cylinder with a hole cut out of the centre. Find the surface area of the entire exterior of the tyre.





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Thus children are more prone to dehydration and hypothermia.