8 Algebra: Brackets

8.1 Expansion of Single Brackets

In this section we consider how to expand (multiply out) brackets to give two or more terms, as shown below:

$$3(x+6) = 3x + 18$$

First we revise *negative numbers* and *order of operations.*

Example 1

Evaluate:

(a) $-6+10$

(b) $-7+(-4)$

(c) $(-6)\times(-5)$

(d) $6\times(4-7)$

(e) $4(8+3)$

(f) $6(8-15)$

(g) $3-(-5)$

(h) $\dfrac{(-2)-(-3)}{-1}$

Solution

(a) $-6+10=4$

(b) $-7+(-4)=-7-4$

$=-11$

(c) $(-6)\times(-5)=30$

(d) $6\times(4-7)=6\times(-3)$

$=-18$

(e) $4(8+3)=4\times11$

$=44$

(f) $6(8-15)=6\times(-7)$

$=-42$

(g) $3-(-5)=3+5$

$=8$

(h) $\dfrac{(-2)-(-3)}{-1}=\dfrac{(-2)+3}{-1}$

$=\dfrac{1}{-1}$

$=-1$

When a bracket is expanded, *every term* inside the bracket must be multiplied by the number outside the bracket. Remember to think about whether each number is positive or negative!

Example 2

Expand $3(x+6)$ using a table.

Solution

$\times$	x	6
3	$3x$	18

From the table,

$$3(x+6)=3x+18$$

Example 3

Expand $4(x-7)$.

Solution

$4(x-7)=4\times x-4\times 7$

$=4x-28$

Remember that every term inside the bracket must be multiplied by the number outside the bracket.

Example 4

Expand $x(8-x)$.

Solution

$$x(8-x) = x \times 8 - x \times x$$
$$= 8x - x^2$$

Example 5

Expand $(-3)(4-2x)$.

Solution

$$(-3)(4-2x) = (-3) \times 4 - (-3) \times 2x$$
$$= -12 - (-6x)$$
$$= -12 + 6x$$

Exercises

1. Calculate:

(a)	$-6+17$	(b)	$6-14$	(c)	$-6-5$
(d)	$6-(-9)$	(e)	$-11-(-4)$	(f)	$(-6)\times(-4)$
(g)	$8\times(-7)$	(h)	$88\div(-4)$	(i)	$6(8-10)$
(j)	$5(3-10)$	(k)	$7(11-4)$	(l)	$(-4)(6-17)$

2. Copy and complete the following tables, and write down each of the expansions:

(a)

×	x	2
4		

$4(x+2) =$

(b)

×	x	-7
5		

$5(x-7) =$

(c)

×	x	3
4		

$4(x+3) =$

(d)

×	$2x$	5
5		

$5(2x+5) =$

3. Expand:

(a) $4(x+6)$ (b) $3(x-4)$

(c) $5(2x+6)$ (d) $7(3x-4)$

(e) $3(2x+4)$ (f) $8(3x-9)$

(g) $(-2)(x-4)$ (h) $(-3)(8-2x)$

(i) $5(3x-4)$ (j) $9(2x+8)$

4. Jordan writes $3(4x-8)=12x-8$.

Explain why his expansion is *not* correct.

5. Copy and complete the following tables and write down each of the expansions:

(a)

×	x	-2
x		

$x(x-2)=$

(b)

×	x	$-y$
x		

$x(x-y)=$

6. Copy the following expansions, filling in the missing terms:

(a) $4x(x+8)=4x^2+\ ?$ (b) $(-3)(2x-7)=\ ?\ +21$

(c) $4x(x-9)=4x^2-\ ?$ (d) $6x(x-7)=6x^2-\ ?$

(e) $3x(x-y)=3x^2-\ ?$ (f) $(-4x)(2x+8)=\ ?\ -32x$

7. Expand:

(a) $x(x-7)$ (b) $x(8-2x)$

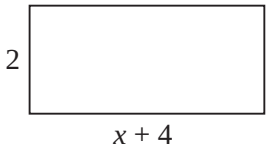
(c) $6x(x+2)$ (d) $4x(3x-5)$

(e) $x(x+y)$ (f) $x(4y-3x)$

(g) $2x(2x+3y)$ (h) $5x(2y-1)$

8. Write down expressions for the area of each of these rectangles, and then expand the brackets:

(a)



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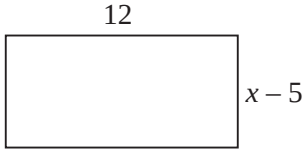

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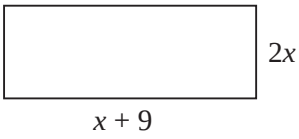

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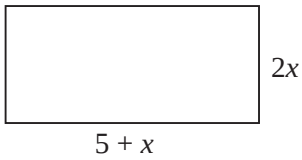

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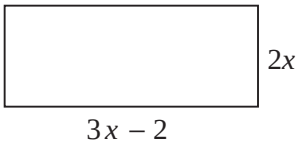
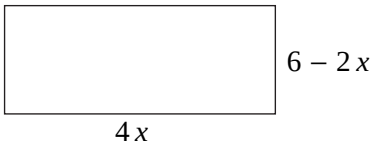


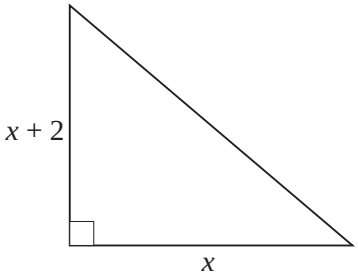
(f)



9. Write down an expression for the area of this triangle, that:

   (a) contains brackets,

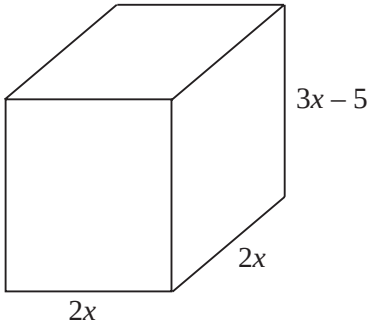
   (b) does *not* contain brackets.



10 Write down an expression for the volume of this cuboid, that:

   (a) contains brackets,

   (b) does *not* contain brackets.



8.2 Linear Equations

Expanding a bracket will usually be the first step when solving an equation like

$$4(x+3)=20$$

Example 1

Solve

$$5(x-3)=35$$

Solution

$$5(x-3) = 35$$

Expanding brackets gives: $5x-15 = 35$

Adding 15 to both sides gives: $5x = 50$

Dividing by 5 gives: $x = 10$

Example 2

Solve

$$6(x+7)=50$$

Solution

$$6(x+7) = 50$$

Expanding brackets gives: $6x+42 = 50$

Subtracting 42 from both sides gives: $6x = 8$

Dividing by 6 gives: $x = \frac{8}{6}$

$$= 1\frac{1}{3}$$

Example 3

Gilda thinks of a number and adds 7 to it. She then multiplies her answer by 4 and gets 64.

(a) Write down an equation that can be used to calculate the number with which Gilda started.

(b) Solve your equation to give the number.

Solution

(a) Start with x.

Add 7 to give $x+7$

Multiply by 4 to give $4(x+7)$

This expression equals 64, so the equation is $4(x+7)=64$

(b) $4(x+7) = 64$

Expanding brackets gives; $4x+28 = 64$

Subtracting 28 from both sides gives: $4x = 36$

Dividing by 4 gives: $x = \frac{36}{4}$

$$= 9$$

Exercises

1. Solve these equations:

   (a) $2(x+6)=14$ (b) $5(x-8)=40$

   (c) $3(x+5)=12$ (d) $7(x+4)=42$

   (e) $2(x+7)=19$ (f) $3(x-4)=11$

   (g) $5(x-4)=12$ (h) $10(x+7)=82$

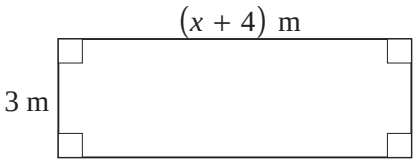
2. Solve these equations:

   (a) $5(2x-7)=8$ (b) $3(3x+6)=27$

   (c) $3(2x+1)=30$ (d) $8(2x-12)=24$

3. A rectangle has sides of length 3 m and $(x+4)$ m.

   Find the value of x, if the area of the rectangle is 18 m^2.

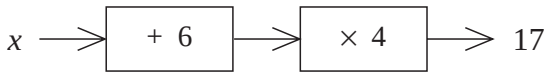


4. Feti chooses a number, adds 7, multiplies the result by 5 and gets the answer 55.

   (a) If x is the number Feti first chose, write down an equation that can be used to determine the number.

   (b) Solve the equation to determine the value of x.

5. The following flow chart is used to form an equation:



   (a) Write down the equation.

   (b) Solve the equation to find the value of x.

6. Solve the following equations:

   (a) $4(7-x)=20$ (b) $3(9-x)=15$

   (c) $6(5-2x)=18$ (d) $5(7-3x)=20$

   (e) $2(10-3x)=17$ (f) $6(9-5x)=4$

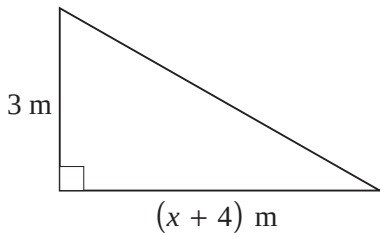
7. Alice thinks of a number, subtracts it from 11 and then multiplies her answer by 5 to get 45. What was the number that Alice started with?

8. Solve the following equations:

(a) $2(x+1)=6(x-3)$ (b) $3(x+4)=11x$

(c) $5(x+4)=2(10x+1)$ (d) $4(7-x)=5(x+2)$

9.



(a) Write down an expression for the area of the triangle.

(b) What is x if the area is $15\ \text{m}^2$?

8.3 Common Factors

As well as being able to remove brackets by expanding expressions, it is also important to be able to write expressions so that they include brackets; this is called *factoring* or *factorisation.*

Example 1

Factorise

$$4x+6$$

Solution

First write each term as a product of factors:

$$4x+6=2\times 2\times x+2\times 3$$

$$4x+6=2(2x+3)$$

[Note that 2 is the only factor common to both terms and is placed outside the brackets.]

Now you can check your answer by expanding it.

Example 2

Factorise

$$18n + 24$$

Solution

$$18n + 24 = \underbrace{2 \times 3} \times \underbrace{3 \times n} + \underbrace{2 \times 2} \times \underbrace{2 \times 3}$$

$$= 6(3n + 4)$$

Note that both 2 and 3 are factors of both terms, and so $2 \times 3 = 6$ is placed outside the brackets.

Example 3

Factorise

$$4x^2 + 6x$$

Solution

$$4x^2 + 6x = 2 \times 2 \times x \times x + 2 \times 3 \times x$$

$$= 2x(2x + 3)$$

Note that both 2 and x are factors of both terms, and so $2 \times x = 2x$ is placed outside the brackets.

Example 4

Factorise

$$5x + 20x^2$$

Solution

$$5x + 20x^2 = 5 \times x + 4 \times 5 \times x \times x$$

$$= 5x(1 + 4x)$$

Note that because 5 and x are factors of both terms, a 1 must be introduced in the bracket when the $5x$ is placed outside the brackets.

You can check the calculation 'backwards':

$$5x(1 + 4x) = 5x \times 1 + 5x \times 4x$$

$$= 5x + 20x^2$$

Example 5

Factorise

$$3xy^2 + 12x^2y$$

Solution

$$3xy^2 + 12x^2y = 3 \times x \times y \times y + 3 \times 4 \times x \times x \times y$$

$$= 3xy(y + 4x)$$

Note that 3, x and y are factors of both terms, and so $3 \times x \times y = 3xy$ is placed outside the brackets.

Exercises

1. Factorise:

   (a) $2x + 4$ (b) $5x + 15$ (c) $6x + 18$

   (d) $5x - 25$ (e) $3x - 21$ (f) $7x + 35$

   (g) $9x - 12$ (h) $15x + 20$ (i) $42x + 15$

2. Factorise:

   (a) $3x^2 + x$ (b) $5x^2 + 10$ (c) $6x - 3x^2$

   (d) $6x^2 - 4x$ (e) $21x^2 + 14x$ (f) $15x - 25x^2$

3. Denise states that

$$4x + 6x^2 = x(4 + 6x)$$

   (a) Is her statement true?

   (b) Describe how it could be improved.

4. For each statement below, decide if it has been fully factorised and if not, complete the factorisation:

   (a) $x^2 + x = x(x + 1)$ (b) $3x^2 + 9x = 3(x^2 + 3x)$

   (c) $5x - 30x^2 = x(5 - 30x)$ (d) $8x^2 - 32x = 4(2x^2 - 8x)$

   (e) $6x^2 - 18x = 3x(2x - 6)$ (f) $15x - 6x^2 = 3(5x - 2x^2)$

5. Explain why the following factorisation is *incorrect*:

$$15x + 24x^2 = 3x(5 + 24x)$$

6. Factorise:

(a) $xy + xz$ (b) $xyz + 3yz$

(c) $4pq - 8qr$ (d) $5xyz + 20uxy$

(e) $5xy - 4py$ (f) $7xy + 12xz$

7. Factorise:

(a) $x^2y + xy^2$ (b) $3x^2y^2 + 6xy^2$

(c) $5x^2y - 35xy$ (d) $22xy + 4xy^2$

(e) $x^2yz + xy^2z$ (f) $x^2y - x^3z$

(g) $x^6y^2 + xy^3$ (h) $x^4y^3 + x^2y^6$

8. (a) Expand $x(x + y + z)$.

(b) Factorise $5x^2 + 2xy + 4xz$.

9. Factorise:

(a) $3x + 9y + 18z$ (b) $4x^2 + 2x + 8xy$

(c) $6x - 3xy + 12xz$ (d) $5xz + 20x - 35xy$

(e) $7x^2 + 14xy - 21xy^2$ (f) $4x + 6xz + 15xy$

10. Factorise:

(a) $4x^2y + 12x^3y^2 + x^2$ (b) $6x^7y^2 - 4x^5y - x^4y^2$

(c) $3x^2y^2 - 4xy^3 + x^4y$ (d) $5x^7y - x^2y^3 + 4x^3z$

8.4 Expansion of Two Brackets

When two brackets are multiplied together, for example,

$$(x+2)(x+3)$$

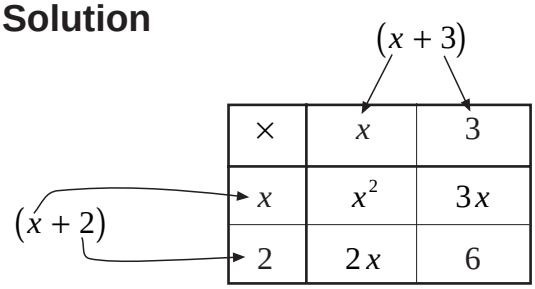
every term in the *first* bracket must be multiplied by *every* term in the *second* bracket.

Example 1

Use a table to determine

$$(x+2)(x+3)$$

Solution



The multiplication table is formed using the two brackets.

The contents of the table give the expansion.

$$(x+2)(x+3) = x^2+3x+2x+6 \quad \text{or} \quad \begin{array}{r} x^2+3x \\ +2x+6 \\ \hline = x^2+5x+6 \end{array}$$

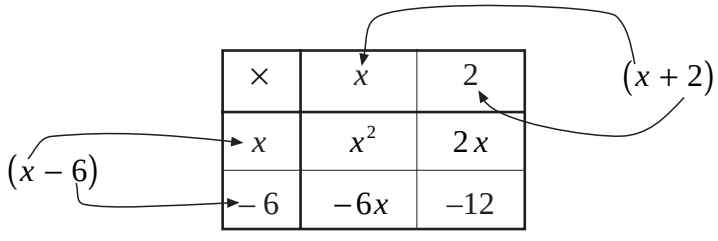
$$= x^2+5x+6$$

Example 2

Use a table to determine

$$(x-6)(x+2)$$

Solution



So,

$$(x-6)(x+2) = x^2+2x-6x-12 \quad \text{or} \quad \begin{array}{r} x^2+2x \\ -6x-12 \\ \hline = x^2-4x-12 \end{array}$$

$$= x^2-4x-12$$

An alternative method for expanding two brackets is shown in the next example.

Example 3

Determine

$$(x+2)(x-7)$$

Solution

$$\begin{aligned}(x+2)(x-7) &= x(x-7)+2(x-7)\\ &= x^2-7x+2x-14 \qquad \text{or} \qquad x^2-7x\\ &= x^2-5x-14 \qquad\qquad\qquad\quad +2x-14\\ &\qquad\qquad\qquad\qquad\qquad\quad = x^2-5x-14\end{aligned}$$

Note how each term in the first bracket multiplies the whole of the second bracket.

Exercises

1. Copy and complete the following tables and write down each of the expansions:

(a)

×	x	5
x		
4		

$(x+4)(x+5)$

(b)

×	x	-7
x		
4		

$(x+4)(x-7)$

(c)

×	x	4
x		
-1		

$(x-1)(x+4)$

(d)

×	x	-5
x		
-2		

$(x-2)(x-5)$

2. Expand:

(a) $(x+3)(x+4)$

(b) $(x-2)(x+5)$

(c) $(x-5)(x-1)$

(d) $(x+7)(x-3)$

(e) $(x+2)(x-3)$

(f) $(x+4)(x-1)$

3. Expand:

(a) $(x-1)(x+1)$ (b) $(x+2)(x-2)$

(c) $(x-5)(x+5)$ (d) $(x-7)(x+7)$

How are the answers to this question different from the others you have done?

4. Explain what is wrong with this statement:

$$(x+5)^2 = x^2 + 25$$

5. Expand:

(a) $(x+1)^2$ (b) $(x-1)^2$

(c) $(x+3)^2$ (d) $(x-5)^2$

6. (a) Copy and complete this table:

$\times$	x	6
$2x$		
1		

(b) What is the expansion of

$$(2x+1)(x+6)\ ?$$

7. Expand:

(a) $(2x+1)(2x+4)$ (b) $(3x+1)(4x+1)$

(c) $(2x-1)(3x+2)$ (d) $(4x-1)(5x+1)$

(e) $(2x+1)^2$ (f) $(4x-3)^2$

8. Write out the following expansions, filling in the missing terms:

(a) $(x+7)(x+6) = x^2 + \ ? \ + 42$ (b) $(x+6)^2 = x^2 + \ ? \ + 36$

(c) $(x-2)(x-5) = x^2 + \ ? \ + 10$ (d) $(x-1)(2x+1) = 2x^2 - x - \ ?$

(e) $(x+3)(2x+1) = \ ? \ + 7x + 3$ (f) $(x-7)^2 = x^2 - \ ? \ + 49$

9. Explain what is wrong with this statement:

$$(x+4)(x-5) = x^2 - 20$$

10. Write out the following expansions, filling in the missing terms:

(a) $(x + ?)(x - 1) = x^2 + x - 2$

(b) $(x + 4)(x - ?) = x^2 - 2x - 24$

(c) $(2x + 3)(x + ?) = 2x^2 + 9x + ?$

(d) $(x - ?)(x + 5) = x^2 - 2x - ?$

(e) $(x + ?)(x + ?) = x^2 + 4x + 4$

(f) $(x + ?)(x + ?) = x^2 + 6x + 8$

11. The following example shows how to determine $(x+1)^3$.

$\times$	x	1
x	x^2	x
1	x	1

$$\begin{aligned}(x+1)^2 &= x^2 + x + x + 1\\ &= x^2 + 2x + 1\end{aligned}$$

$\times$	x^2	$2x$	1
x	x^3	$2x^2$	x
1	x^2	$2x$	1

$$\begin{aligned}(x+1)^3 &= (x+1)(x^2 + 2x + 1)\\ &= x^3 + 2x^2 + x + x^2 + 2x + 1\\ &= x^3 + 3x^2 + 3x + 1\end{aligned}$$

Use the same method to determine:

(a) $(x+1)^4$,

(b) $(x+1)^5$.

Compare your answers with Pascal's Triangle and describe any connections that you see.