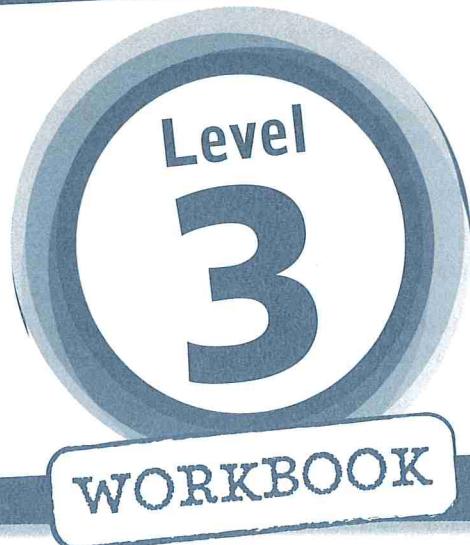


Edexcel Award in Algebra



ANSWERS

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Answers

1 Algebraic manipulation

1.1 Expanding two brackets

1 a $6x^2 - 15x$

x	x	+ 2
x	x^2	$+2x$
+ 3	$+3x$	+ 6

$x^2 + 2x + 3x + 6 = x^2 + 5x + 6$

c $2x^2 - 10x + 3x - 15 = 2x^2 - 7x - 15$

d $6x^2 - 8xy - 15xy + 20y^2 = 6x^2 - 23xy + 20y^2$

2 a $2x^2 + 8x$

b $18x^2 - 30x$

c $10x^2 - 10xy$

d $x^2 + 9x + 20$

e $x^2 + 10x + 21$

f $x^2 + 5x - 14$

g $x^2 - 25$

h $2x^2 + x - 3$

i $6x^2 - x - 2$

j $10x^2 - 31x + 15$

k $12x^2 + 13x - 14$

l $18x^2 + 39xy + 20y^2$

m $35x^2 + 14xy - 15x - 6y$

n $6x^2 - 16x - 9xy + 24y$

1.2 Factorising expressions

1 a $3x^2(5y^2 + 3x^2)$

b $(2x - 5y)(2x + 5y)$

c $b = 3, ac = -10$

$x^2 + 3x - 10 = x^2 + 5x - 2x - 10$

$= x(x + 5) - 2(x + 5)$

$= (x + 5)(x - 2)$

d $b = -11, ac = -60$

Two numbers are -15 and 4

$6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$

$= 3x(2x - 5) + 2(2x - 5)$

$= (3x + 2)(2x - 5)$

2 a $2x^3y^3(3x - 5y)$

b $7a^3b^2(3b^3 + 5a^2)$

c $5x^2y^2(5 - 2x + 3y)$

3 a $(x + 3)(x + 4)$

b $(x + 7)(x - 2)$

c $(x - 5)(x - 6)$

d $(x - 8)(x + 3)$

e $(x - 9)(x + 2)$

f $(x + 5)(x - 4)$

g $(x - 8)(x + 5)$

h $(x + 7)(x - 4)$

4 a $(6x - 7y)(6x + 7y)$

b $(2x - 9y)(2x + 9y)$

c $2(3a - 10bc)(3a + 10bc)$

5 a $(x - 1)(2x + 3)$

b $(3x + 1)(2x + 5)$

c $2(3x - 2)(2x - 5)$

d $(2x + 1)(x + 3)$

e $(3x - 1)(3x - 4)$

f $(5x + 3)(2x + 3)$

1.3 Using index laws

1 a 1

b $\sqrt{9} = 3$

c $(\sqrt[3]{27})^2 = 3^2 = 9$

d $\frac{1}{4^2} = \frac{1}{16}$

e $3x^3$

f $\frac{x^8}{x^4} = x^4$

g $\frac{x^5}{x^2} = x^{\frac{5}{2}}$

h $\frac{12x^2}{8x^6} = \frac{3}{2x^4}$

2 a 1

b 1

c 1

3 a 7

b 4

c 5

d 2

4 a 125

b 32

c 343

d 8

5 a $\frac{1}{25}$

b $\frac{1}{64}$

c $\frac{1}{32}$

d $\frac{1}{36}$

6 a $\frac{3x^3}{2}$

b $5x^2$

c $3x$

d $\frac{y}{2x^2}$

e $y^{\frac{1}{2}}$

f c^{-3}

g $2x^6$

h x

7 a $\frac{1}{2}$

b $\frac{1}{9}$

c $\frac{8}{3}$

d $\frac{1}{4}$

e $\frac{4}{3}$

f $\frac{16}{9}$

1.4 Algebraic fractions

1 a $\frac{2x(x - 2)}{6x(2 + x)} = \frac{x - 2}{3(x + 2)}$

b $\frac{(x + 3)(x - 7)}{(2x + 3)(x + 3)} = \frac{x - 7}{2x + 3}$

c $\frac{2x}{6} + \frac{6x + 3}{6} = \frac{8x + 3}{6}$

d $\frac{2x + 2}{(x - 3)(x + 1)} - \frac{5x - 15}{(x - 3)(x + 1)} = \frac{-3x + 17}{(x - 3)(x + 1)}$

2 a $\frac{2(x + 2)}{x - 1}$

b $\frac{x}{x - 1}$

c $\frac{x + 2}{x}$

d $\frac{x}{x + 5}$

e $\frac{x + 3}{x}$

f $\frac{x}{x - 5}$

3 a $\frac{13x}{15}$

b $\frac{11x + 5}{10}$

c $\frac{x}{28}$

d $\frac{x}{12}$

e $\frac{11x + 4}{12}$

f $\frac{7x + 13}{20}$

4 a $\frac{5x + 11}{(x + 3)(x + 1)}$

b $\frac{3(x + 1)}{x(x + 3)}$

c $\frac{x - 8}{x(x + 4)}$

d $\frac{2(x - 3)}{(x + 1)(x - 1)}$

e $\frac{5(x + 2)}{(2x - 3)(x + 1)}$

f $\frac{5x - 4}{(x + 1)(x - 2)}$

5 a $\frac{3x + 4}{x + 7}$

b $\frac{2x + 3}{3x - 2}$

c $\frac{2 - 5x}{2x - 3}$

d $\frac{3x + 1}{x + 4}$

1.5 Completing the square

1 a $(x + 3)^2 - 2 - 9 = (x + 3)^2 - 11$

b $2(x^2 - \frac{5}{2}x + \frac{1}{2}) = 2[(x - \frac{5}{4})^2 + \frac{1}{2} - \frac{25}{16}] = 2[(x - \frac{5}{4})^2 - \frac{17}{16}] = 2(x - \frac{5}{4})^2 - \frac{17}{8}$

2 a $(x + 2)^2 - 1$

b $(x - 5)^2 - 28$

c $(x - 4)^2 - 16$

d $(x + 3)^2 - 9$

e $(x - 1)^2 + 6$

f $(x + \frac{3}{2})^2 - \frac{17}{4}$

3 a $2(x - 2)^2 - 24$

b $4(x - 1)^2 - 20$

c $3(x + 2)^2 - 21$

d $2(x + \frac{3}{2})^2 - \frac{25}{2}$

e $3(x - \frac{1}{3})^2 - \frac{1}{3}$

f $3(x + \frac{5}{6})^2 + \frac{11}{12}$

Don't forget!

* four

* $ax^2 + bx + c$

* $b; ac$

* the difference of two squares; $(x - y)(x + y)$

* a^{m+n}

* a^{m-n}

* a^{mn}

* 1

* common denominator; equivalent

* $p(x + q)^2 + r$

Exam-style questions

1 a $3x^2 - 7x - 6$

b $6x^2y^2(2x + 5y^3)$

c x^2

2 a x^{-2}

b $(x - 5)(x + 7)$

c $(2x - 5y)(2x + 5y)$

3 $(x + 1\frac{1}{2})^2 - 7\frac{1}{4}$

4 $\frac{x + 2}{2x + 3}$

2 Formulae

2.1 Substitution

1 a $2 \times 8 + (-6) = 16 - 6 = 10$

b $8 + (-6) \times \frac{1}{3} = 8 - 2 = 6$

c $\frac{3 \times 8}{-6} = -4$

d $8^{\frac{1}{3}} - (-6) = 2 + 6 = 8$

2 $C = \frac{5}{9}$ of $(50 - 32)$

$C = \frac{5}{9}$ of 18

$C = 5 \times 18 \div 9$

$C = 10$

- 3** a 7
e -324
4 a -1.6
5 a $-2\frac{2}{3}$
6 610
- b** 3
f -18
c $20\frac{1}{2}$
d 25
- b** 2.7
c 2.8
d -2.4
- b** $-\frac{1}{6}$
c $-1\frac{1}{6}$
d -7

2.2 Changing the subject of a formula

- 1** $v - u = at$
 $t = \frac{v - u}{a}$
- 2** $r = t(2 - \pi)$
 $t = \frac{r}{2 - \pi}$
- 3** $2(t + r) = 5 \times 3t$
 $2t + 2r = 15t$
 $2r = 13t$
 $t = \frac{2r}{13}$
 $t = \frac{5+r}{r-3}$
- 4** $r(t - 1) = 3t + 5$
 $rt - r = 3t + 5$
 $rt - 3t = 5 + r$
 $t(r - 3) = 5 + r$
 $t = \frac{5+r}{r-3}$
- 5** $d = \frac{C}{\pi}$
6 $w = \frac{P - 2l}{2}$
7 $T = \frac{S}{D}$
- 8** $t = \frac{q - r}{p}$
9 $t = \frac{2u}{2a - 1}$
10 $x = \frac{V}{a + 4}$
- 11** $y = 2 + 3x$
12 $a = \frac{3x + 1}{x + 2}$
13 $d = \frac{b - c}{a}$
- 14** $g = \frac{2h + 9}{7 - h}$
15 $e = \frac{1}{x + 7}$

Don't forget!

- * replacing each letter with its value
- * everything else

Exam-style questions

1 $x = \frac{4y - 3}{2 + y}$

3 Surds

3.1 Surds

- 1** $\sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5 \times \sqrt{2} = 5\sqrt{2}$
- 2** $\sqrt{49 \times 3} - 2\sqrt{4 \times 3} = \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$
 $= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3} = 3\sqrt{3}$
- 3** $\sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4} = 7 - 2 = 5$
- 4** a $3\sqrt{5}$ b $5\sqrt{5}$ c $4\sqrt{3}$ d $5\sqrt{7}$
e $10\sqrt{3}$ f $2\sqrt{7}$ g $6\sqrt{2}$ h $9\sqrt{2}$
- 5** a -1 b $9 - \sqrt{3}$ c $10\sqrt{5} - 7$ d $26 - 4\sqrt{2}$
- 6** a $15\sqrt{2}$ b $\sqrt{5}$ c $3\sqrt{2}$ d $\sqrt{3}$
e $6\sqrt{7}$ f $5\sqrt{3}$

3.2 Rationalising the denominator

- 1** a $\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
- b $\frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} = \frac{\sqrt{2} \times 2\sqrt{3}}{12} = \frac{\sqrt{6}}{6}$
- c $\frac{3}{2 + \sqrt{5}} \times \frac{2 - \sqrt{5}}{2 - \sqrt{5}} = \frac{3(2 - \sqrt{5})}{4 + 2\sqrt{5} - 2\sqrt{5} - 5} = \frac{3(2 - \sqrt{5})}{-1}$
 $= -3(2 - \sqrt{5}) = -6 + 3\sqrt{5}$
- 2** a $\frac{\sqrt{5}}{5}$ b $\frac{\sqrt{11}}{11}$ c $\frac{2\sqrt{7}}{7}$ d $\frac{\sqrt{2}}{2}$
e $\sqrt{2}$ f $\sqrt{5}$ g $\frac{\sqrt{3}}{3}$ h $\frac{1}{3}$
- 3** a $\frac{3 + \sqrt{5}}{4}$ b $\frac{2(4 - \sqrt{3})}{13}$ c $\frac{6(5 + \sqrt{2})}{23}$

Don't forget!

- * the square root of a number that is not a square number
- * $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc.
- * $\sqrt{a} \times \sqrt{b}$
- * $\frac{\sqrt{a}}{\sqrt{b}}$
- * denominator
- * \sqrt{b}
- * $b - \sqrt{c}$

Exam-style questions

- 1** $2\sqrt{5}$ **2** $9 - 4\sqrt{2}$ **3** $10 + 5\sqrt{3}$ **4** $\frac{3\sqrt{5}}{5}$ **5** $7\sqrt{2}$

4 Quadratic equations

4.1 Solving by factorisation

- 1** a $5x^2 - 15x = 0$
 $5x(x - 3) = 0$
So $5x = 0$ or $x - 3 = 0$
 $x = 0$ or $x = 3$
- b $(x + 4)(x + 3) = 0$
So $x + 4 = 0$ or $x + 3 = 0$
 $x = -4$ or $x = -3$
- c $(3x + 4)(3x - 4) = 0$
So $3x + 4 = 0$ or $3x - 4 = 0$
 $x = -1\frac{1}{3}$ or $x = 1\frac{1}{3}$
- d $(2x + 3)(x - 4) = 0$
So $2x + 3 = 0$ or $x - 4 = 0$
 $x = -1\frac{1}{2}$ or $x = 4$
- 2** a $x = 0$ or $x = -\frac{2}{3}$
b $x = 0$ or $x = \frac{3}{4}$
- c $x = -5$ or $x = -2$
d $x = 2$ or $x = 3$
- e $x = -1$ or $x = 4$
f $x = -5$ or $x = 2$
- g $x = 4$ or $x = 6$
h $x = -6$ or $x = 6$
- i $x = -7$ or $x = 4$
j $x = 3$
- k $x = -\frac{1}{2}$ or $x = 4$
l $x = -\frac{2}{3}$ or $x = 5$
- 3** a $x = -2$ or $x = 5$
b $x = -1$ or $x = 3$
c $x = -8$ or $x = 3$
e $x = -5$ or $x = 5$
g $x = -3$ or $x = 2\frac{1}{2}$
- d $x = -6$ or $x = 7$
f $x = -4$ or $x = 7$
h $x = -\frac{1}{3}$ or $x = 2$

4.2 Solving by completing the square

- 1** $(x + 3)^2 + 4 - 9 = 0$
 $(x + 3)^2 - 5 = 0$
 $(x + 3)^2 = 5$
 $x + 3 = \pm\sqrt{5}$
 $x = -3 \pm\sqrt{5}$
 $x = -3 + \sqrt{5}$ or $x = -3 - \sqrt{5}$
- 2** $2[x^2 - \frac{7}{2}x + 2] = 0$
 $2[(x - \frac{7}{4})^2 + 2 - \frac{49}{16}] = 0$
 $(x - \frac{7}{4})^2 - \frac{17}{16} = 0$
 $(x - \frac{7}{4})^2 = \frac{17}{16}$
 $x - \frac{7}{4} = \pm\sqrt{\frac{17}{16}}$
 $x - \frac{7}{4} = \pm\frac{1}{4}\sqrt{17}$
 $x = \frac{7 + \sqrt{17}}{4}$ or $x = \frac{7 - \sqrt{17}}{4}$
- 3** a $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$
b $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$
c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$
d $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$
e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$
f $x = \frac{-3 + \sqrt{89}}{10}$ or $x = \frac{-3 - \sqrt{89}}{10}$
- 4** a $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$
b $x = \frac{-3 + \sqrt{23}}{2}$ or $x = \frac{-3 - \sqrt{23}}{2}$
c $x = \frac{5 + \sqrt{13}}{2}$ or $x = \frac{5 - \sqrt{13}}{2}$

4.3 Solving by using the formula

$$\begin{aligned} \text{1} \quad x &= \frac{-(6) \pm \sqrt{(6)^2 - 4 \times 1 \times 4}}{2 \times 1} \\ &= \frac{-6 \pm \sqrt{36 - 16}}{2} \\ &= \frac{-6 \pm \sqrt{20}}{2} \\ &= \frac{-6 \pm \sqrt{4 \times 5}}{2} \\ &= \frac{-6 + 2\sqrt{5}}{2} \text{ or } x = \frac{-6 - 2\sqrt{5}}{2} \\ &= -3 + \sqrt{5} \text{ or } x = -3 - \sqrt{5} \end{aligned}$$

- 2** $a = 3, b = -7, c = -2$
 $x = \frac{7 \pm \sqrt{49 - 4 \times 3 \times -2}}{2 \times 3}$
 $x = \frac{7 \pm \sqrt{49 + 24}}{6}$
 $x = \frac{7 \pm \sqrt{73}}{6}$
 $x = \frac{7 + \sqrt{73}}{6}$ or $x = \frac{7 - \sqrt{73}}{6}$
- 3** **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$
b $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$
- 4** **a** $x = \frac{7 + \sqrt{17}}{8}$ or $x = \frac{7 - \sqrt{17}}{8}$
b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

Don't forget!

- * two; $b; ac$
- * $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- * negative

Exam-style questions

- 1** $x = \frac{7 + \sqrt{41}}{2}$ or $x = \frac{7 - \sqrt{41}}{2}$
- 2** $x = -1\frac{2}{3}$ or $x = 2$
- 3** $x = \frac{-3 + \sqrt{89}}{20}$ or $x = \frac{-3 - \sqrt{89}}{20}$

5 Roots of quadratic equations**5.1 The role of the discriminant**

- 1** $a = 3, b = 7, c = 5$
 $b^2 - 4ac = 7^2 - 4 \times 3 \times 5 = 49 - 60 = -11$; no real roots
- 2** $b^2 - 4ac = 0$
 $a = 1, b = 4, c = p$
 $b^2 - 4ac = 4^2 - 4 \times 1 \times p$
 $16 - 4p = 0$
 $4p = 16$
 $p = 4$
- 3** $b^2 - 4ac < 0$
 $a = h, b = 3, c = -7$
 $b^2 - 4ac = 3^2 - 4 \times h \times -7 = 9 + 28h$
 $9 + 28h < 0$
 $28h < -9$
 $h < -\frac{9}{28}$
- 4** no real roots
- 5** two real and distinct roots
- 6** two real and equal roots
- 7** no real roots
- 8** $q = \pm 8$
- 9** $q = \pm 6\sqrt{2}$
- 10** $r = \pm 5$
- 11** $t > -\frac{4}{3}$

5.2 The sum and product of the roots of a quadratic equation

- 1** $a = 2, b = 6, c = -5$
Sum = $-\frac{b}{a} = -\frac{6}{2} = -3$
Product = $\frac{c}{a} = \frac{-5}{2} = -2.5$
- 3** sum = 11, product = 30
- 5** sum = 0, product = $-\frac{16}{9}$
- 7** $x^2 + 2x - 8 = 0$
- 9** $2x^2 + 17x - 9 = 0$
- 2** $-\frac{b}{a} = -7, \frac{c}{a} = 10$
 $x^2 - (-7)x + (10) = 0$
 $x^2 + 7x + 10 = 0$
- 4** sum = -1.6, product = -4.2
- 6** sum = $-\frac{1}{6}$, product = $-\frac{5}{2}$
- 8** $3x^2 + x - 2 = 0$
- 10** $2x^2 + 3x - 29 = 0$

Don't forget!

- * $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- * discriminant
- * two real and distinct roots
- * $-\frac{b}{a}$
- * $x^2 - (-\frac{b}{a})x + \frac{c}{a} = 0$

Exam-style questions

- 1** $\pm 8\sqrt{3}$
- 2** $2x^2 + 5x + 9 = 0$

6 Simultaneous equations**6.1 Solving simultaneous linear equations using elimination**

- 1** $2x = 4$
 $x = 2, y = -1$
- 3** $x = 1, y = 4$
- 5** $x = 2, y = -5$
- 7** $x = 6, y = -1$
- 2** $6x = 18$
 $x = 3, y = 5$
- 4** $x = 3, y = -2$
- 6** $x = 3, y = -\frac{1}{2}$
- 8** $x = -2, y = 5$

6.2 Solving simultaneous linear equations using substitution

- 1** $5x + 3(2x + 1) = 14$
 $5x + 6x + 3 = 14$
 $11x = 11$
 $x = 1, y = 3$
- 3** $x = 9, y = 5$
- 5** $x = \frac{1}{2}, y = 3\frac{1}{2}$
- 7** $x = -4, y = 5$
- 9** $x = \frac{1}{4}, y = 1\frac{3}{4}$
- 2** $4x + 3(2x - 16) = -3$
 $4x + 6x - 48 = -3$
 $10x = 45$
 $x = 4.5, y = -7$
- 4** $x = -2, y = -7$
- 6** $x = \frac{1}{2}, y = 3$
- 8** $x = -2, y = -5$
- 10** $x = -2, y = \frac{5}{2}$

6.3 Solving simultaneous equations where one is quadratic

- 1** $x^2 + (x + 1)^2 = 13$
 $x^2 + x^2 + 2x + 1 - 13 = 0$
 $2x^2 + 2x - 12 = 0$
 $2(x^2 + x - 6) = 0$
 $(x + 3)(x - 2) = 0$
 $x = -3$ or $x = 2$
when $x = -3, y = -2$
when $x = 2, y = 3$
- 2** $x = \frac{5 - 3y}{2}$
 $2y^2 + \frac{y(5 - 3y)}{2} = 12$
 $2y^2 + \frac{5y - 3y^2}{2} - 12 = 0$
 $4y^2 + 5y - 3y^2 - 24 = 0$
 $y^2 + 5y - 24 = 0$
 $(y + 8)(y - 3) = 0$
 $y = -8$ or $y = 3$
when $y = -8, x = 14\frac{1}{2}$
when $y = 3, x = -2$
- 3** $x = 0, y = 5$
 $x = -5, y = 0$
- 5** $x = -2, y = -4$
 $x = 2, y = 4$
- 7** $x = \frac{1 + \sqrt{5}}{2}, y = \frac{-1 + \sqrt{5}}{2}$
 $x = \frac{1 - \sqrt{5}}{2}, y = \frac{-1 - \sqrt{5}}{2}$
- 4** $x = -\frac{8}{3}, y = -\frac{19}{3}$
 $x = 3, y = 5$
- 6** $x = \frac{5}{2}, y = 6$
 $x = 3, y = 5$
- 8** $x = \frac{-1 + \sqrt{7}}{2}, y = \frac{3 + \sqrt{7}}{2}$
 $x = \frac{-1 - \sqrt{7}}{2}, y = \frac{3 - \sqrt{7}}{2}$

Don't forget!

- * elimination; substitution
- * two

Exam-style questions

- 1** $x = 4, y = -2$
 $x = -3\frac{1}{2}, y = 2\frac{1}{2}$
- 2** $x = 2\frac{1}{2}, y = \frac{1}{2}$

7 Arithmetic series**7.1 General (nth) term of arithmetic series**

- 1** First term = $4 \times 1 + 1 = 5$ Fourth term = $4 \times 4 + 1 = 17$
Second term = $4 \times 2 + 1 = 9$ Fifth term = $4 \times 5 + 1 = 21$
Third term = $4 \times 3 + 1 = 13$ First 5 terms are 5, 9, 13, 17, 21

2 $5n - 2 = 73$

$5n = 75$

$n = 15$

3 $a = 3, d = 5$

n th term $= 3 + (n - 1) \times 5$

$= 3 + 5n - 5$

$= 5n - 2$

4 $8 + 13 + 18$

5 $3n + 2; 62$

6 $17 - 2n; -3$

7 $82; 402$

8 $8; -97$

9 25

10 53

11 5

12 first term $= 1$, common difference $= 3$

7.2 The sum of an arithmetic series

1 $a = 1, d = 4, n = 30$

$S_n = \frac{30}{2} [2 \times 1 + (30 - 1) \times 4]$

$S_n = 15 \times (2 + 29 \times 4)$

$S_n = 1770$

3 $S_n = 352, a = 7, d = 2$

$352 = \frac{n}{2} [2 \times 7 + (n - 1) \times 2]$

$704 = n(14 + 2n - 2)$

$704 = 2n^2 + 12n$

$2n^2 + 12n - 704 = 0$

$n^2 + 6n - 352 = 0$

$(n + 22)(n - 16) = 0$

$n = 16$

first term $= -2$; common difference $= 5$

5 610

6 1395

7 -5350

8 290

9 341

10 1370

11 488

12 10

13 first term $= 2$; common difference $= 3$

Don't forget!

* sequence

* n th term

* the same amount

* $a + (n - 1)d$

* $\frac{n}{2}[2a + (n - 1)d]$

* $\frac{n}{2}(a + L)$

Exam-style questions

1 a first term $= 60$; common difference $= -7$

b -1245

8 Coordinate geometry

8.1 The equation of a line

1 $y = -\frac{1}{2}x + 3$

$2y = -x + 6$

$x + 2y - 6 = 0$

3 $m = 3$

$y = 3x + c$

$13 = 3 \times 5 + c$

$13 = 15 + c$

$c = -2$

$y = 3x - 2$

5 a $m = 3, c = 5$

c $m = 2, c = -\frac{3}{2}$

e $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$

6 $y = 5x$

$y = -3x + 2$

$y = 4x - 7$

7 a $x + 2y + 14 = 0$

c $2x - 3y + 12 = 0$

8 $y = 4x - 3$

9 $y = -\frac{2}{3}x + 7$

10 a $y = 2x - 3$

c $y = 5x - 2$

2 $3y = 2x - 4$

$y = \frac{2}{3}x - \frac{4}{3}$

gradient $= m = \frac{2}{3}$

y-intercept $= c = -\frac{4}{3}$ or $-1\frac{1}{3}$

4 $m = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$

$y = \frac{1}{2}x + c$

4 (or 7) $= \frac{1}{2} \times 2$ (or 8) $+ c$

4 (or 7) $= 1$ (or 4) $+ c$

$c = 3$

$y = \frac{1}{2}x + 3$

b $m = -\frac{1}{2}, c = -7$

d $m = -1, c = 5$

f $m = -5, c = 4$

b $2x - y = 0$

d $6x + 5y + 10 = 0$

b $y = -\frac{1}{2}x + 6$

d $y = -3x + 19$

8.2 Parallel and perpendicular lines

1 $m = 2$

$y = 2x + c$

$9 = 2 \times 4 + c$

$c = 1$

$y = 2x + 1$

2 $m = 2$

$-\frac{1}{m} = -\frac{1}{2}$

$y = -\frac{1}{2}x + c$

$5 = -\frac{1}{2} \times -2 + c = 1 + c$

$c = 4$

$y = -\frac{1}{2}x + 4$

3 $m = \frac{1}{2}$

$-\frac{1}{m} = -2$

$y = -2x + c$

$3 = -2 \times -5 + c$

$c = -7$

$y = -2x - 7$

4 $m = \frac{-1-5}{9-0} = -\frac{6}{9} = -\frac{2}{3}$

$-\frac{1}{m} = \frac{3}{2}$

$y = \frac{3}{2}x + c$

$\left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$

$y = \frac{3}{2}x + c$

$2 = \frac{3}{2} \times \frac{9}{2} + c = \frac{27}{4} + c$

$c = -\frac{19}{4}$

$y = \frac{3}{2}x - \frac{19}{4}$

5 a $y = 3x - 7$

b $y = -2x + 5$

c $y = -\frac{1}{2}x$

d $y = \frac{3}{2}x + 8$

6 a $y = -\frac{1}{2}x + 2$

b $y = 3x + 7$

c $y = -4x + 35$

d $y = \frac{5}{2}x - 8$

7 a $y = -\frac{1}{2}x$

b $y = 2x$

8 a parallel

b neither

c perpendicular

d neither

e neither

f parallel

Don't forget!

* $y = mx + c$

* $ax + by + c = 0$

* $m = \frac{y_2 - y_1}{x_2 - x_1}$

* gradient

* $-\frac{1}{m}$

Exam-style questions

1 a $x + 2y - 4 = 0$

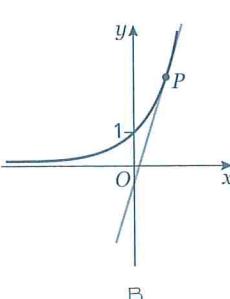
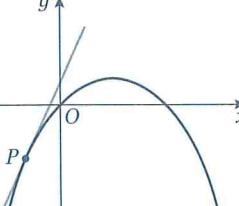
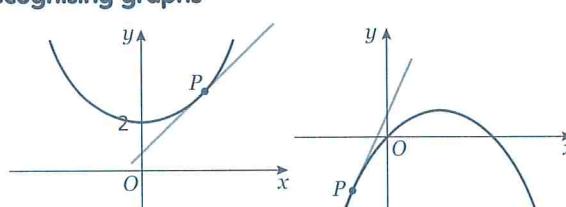
b $x + 2y + 2 = 0$

c $y = 2x$

9 Graphs of functions

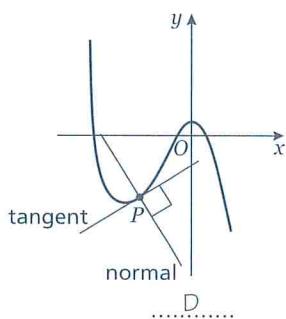
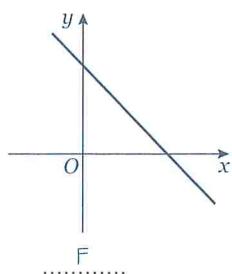
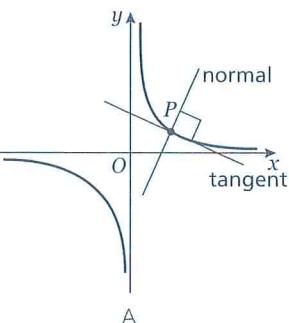
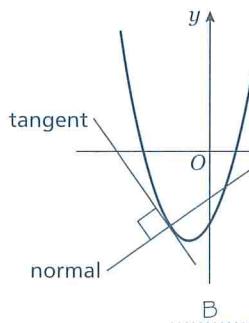
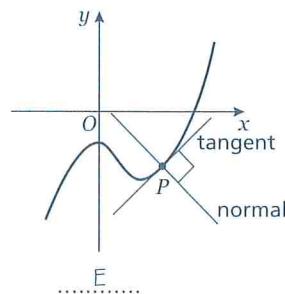
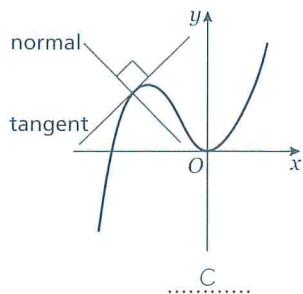
9.1 Recognising graphs

1 a, b



C.....

2 a, b

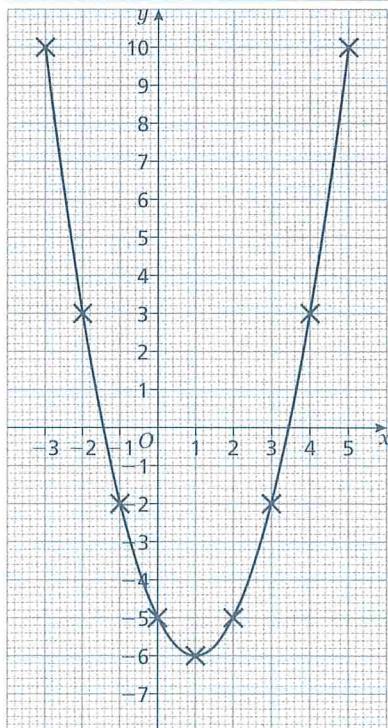


3 B, C, A

9.2 Drawing and using graphs

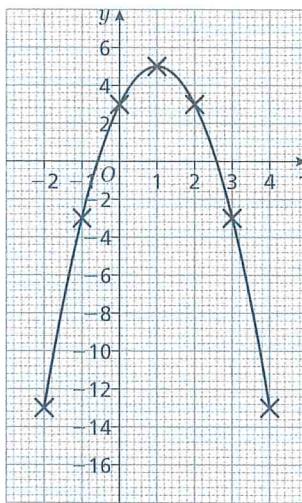
1 a

x	-3	-2	-1	0	1	2	3	4	5
y	10	3	-2	-5	-6	-5	-2	3	10

b $x \approx -1.4$ or -1.5 or $x \approx 3.4$ or 3.5

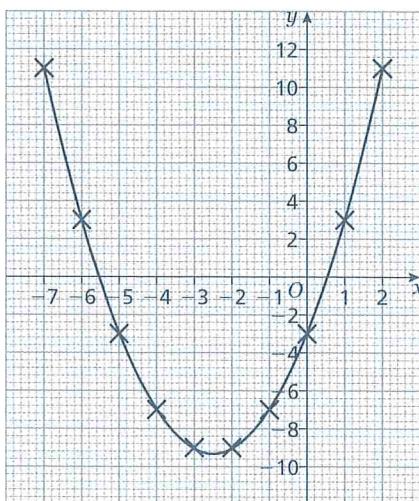
2 a

x	-2	-1	0	1	2	3	4
y	-13	-3	3	5	3	-3	-13

b $x \approx -0.6$ or $x \approx 2.6$

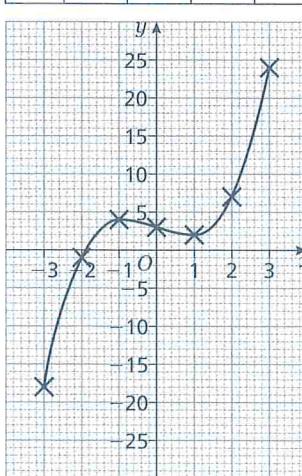
3 a

x	-7	-6	-5	-4	-3	-2	-1	0	1	2
y	11	3	-3	-7	-9	-9	-7	-3	3	11

b $x \approx -5.5$ or -5.6 or $x \approx 0.5$ or 0.6

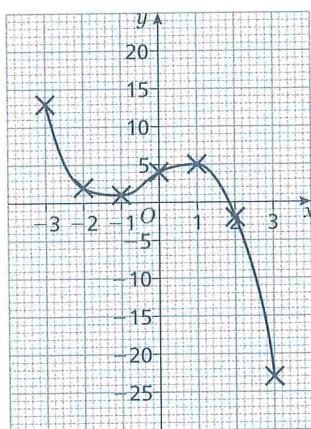
4 a

x	-3	-2	-1	0	1	2	3
y	-18	-1	4	3	2	7	24

b $x \approx -1.4$, $x = 0$ or $x \approx 1.4$

5 a

x	-3	-2	-1	0	1	2	3
y	13	2	1	4	5	-2	-23

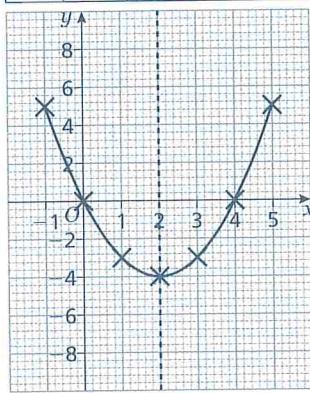


b $x \approx 1.8$

c $x = -2, x = -1$ or $x = 2$

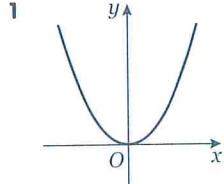
6 a, b

x	-1	0	1	2	3	4	5
y	5	0	-3	-4	-3	0	5



b $x = 2$

9.3 Sketching graphs



2 When $x = 0, y = 0^2 - 0 - 6 = -6$
 $(0, -6)$

$(x - 3)(x + 2) = 0$

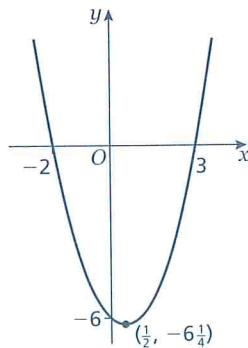
$x = 3$ or $x = -2$

$(-2, 0)$ and $(3, 0)$

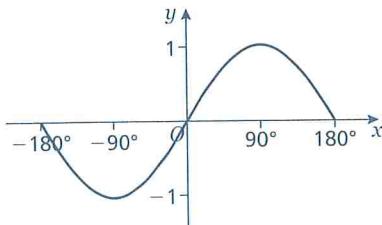
$(x - \frac{1}{2})^2 - 6\frac{1}{4}$

$x = \frac{1}{2}$ and $y = -6\frac{1}{4}$

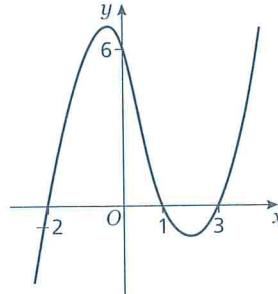
$(\frac{1}{2}, -6\frac{1}{4})$



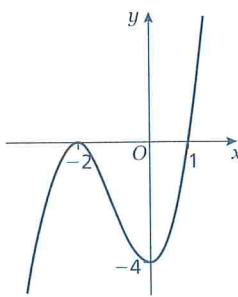
3



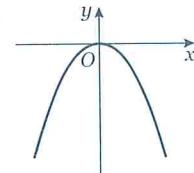
4 $x = 3, 1$ or -2
 $(-2, 0), (1, 0)$ and $(3, 0)$
 $y = -3 \times -1 \times 2 = 6$
 $(0, 6)$



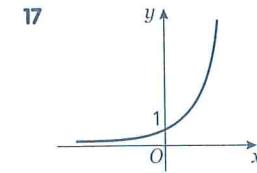
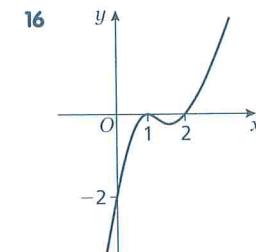
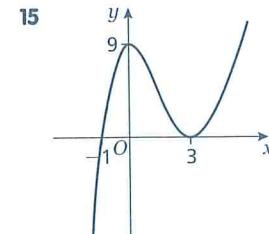
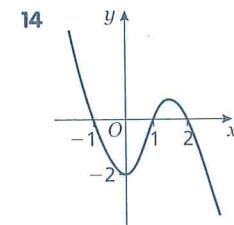
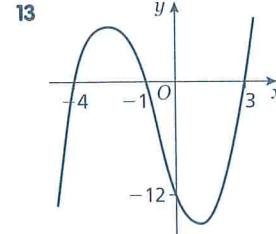
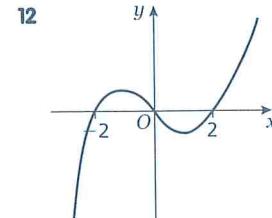
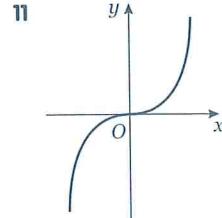
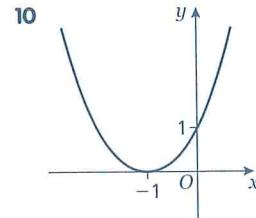
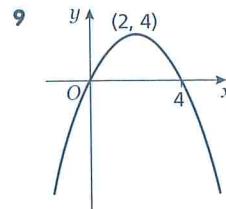
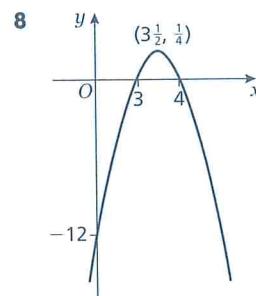
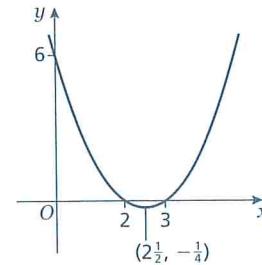
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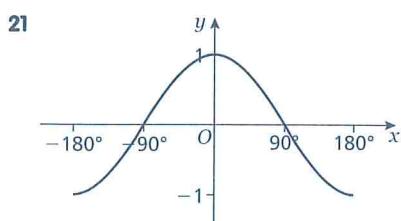
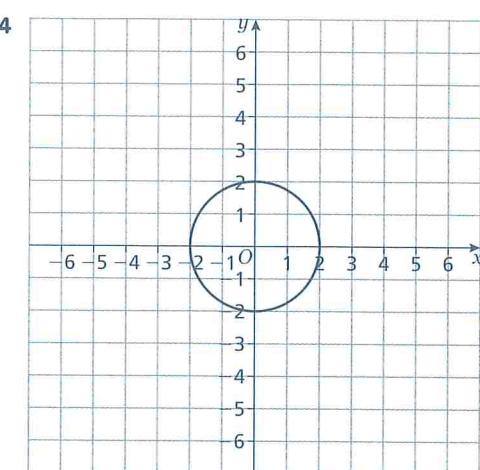
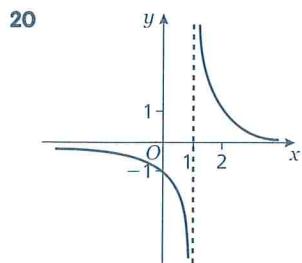
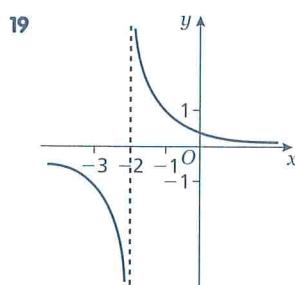
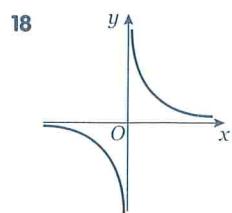


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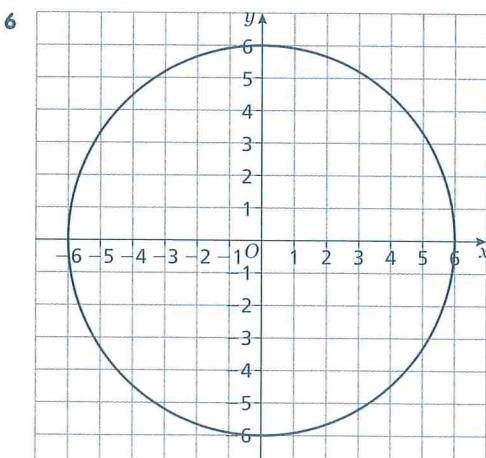
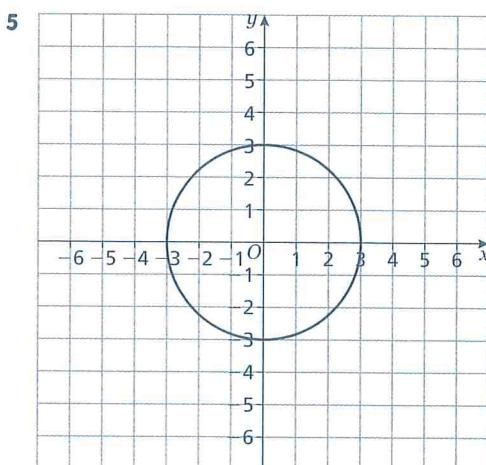
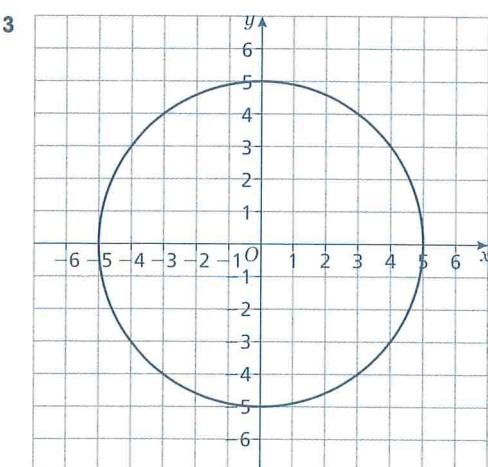
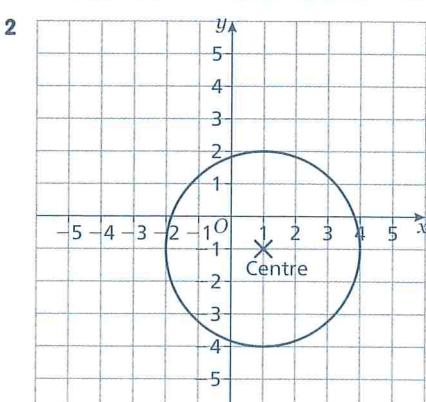
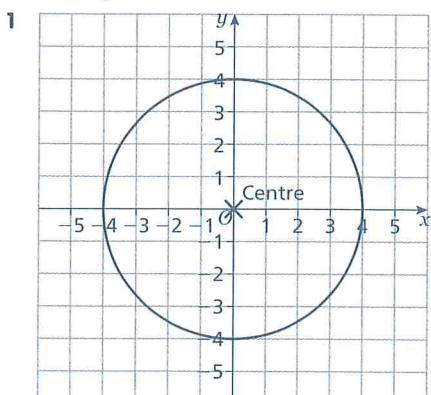


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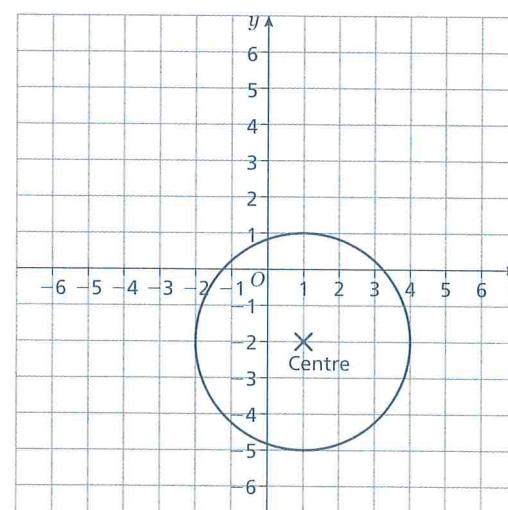


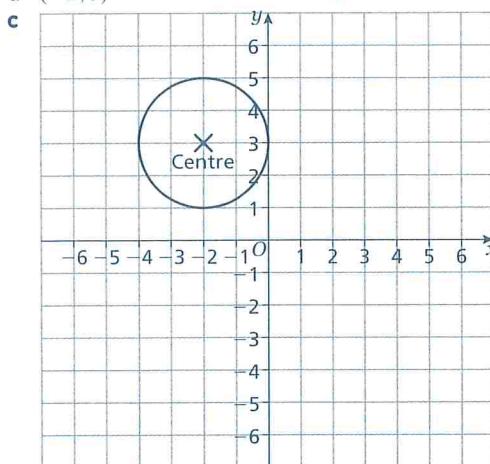
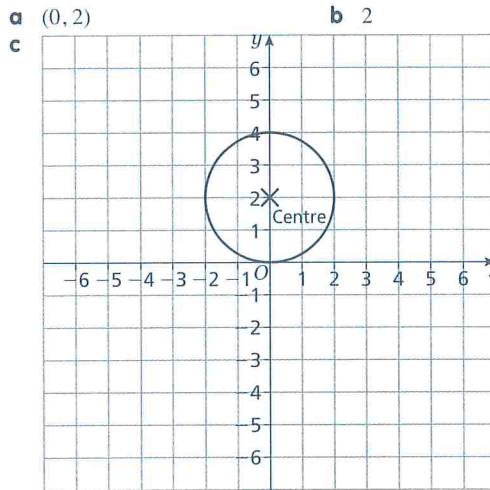


9.4 Graphs of circles



- 7 a (1, -2) b 3



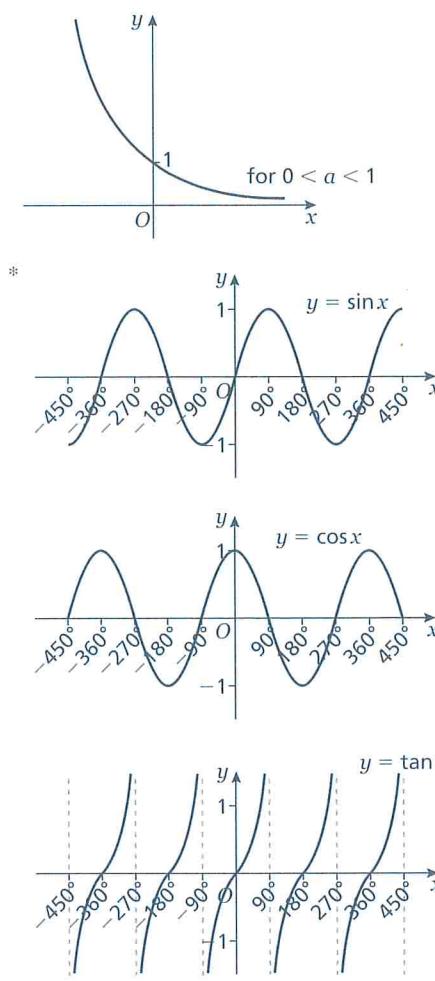
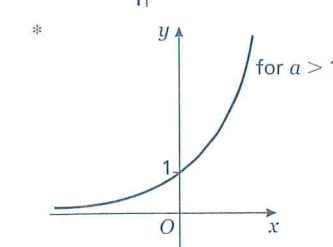
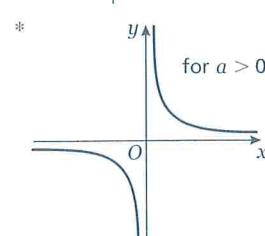
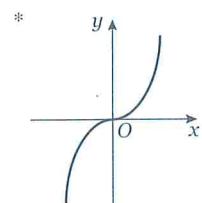
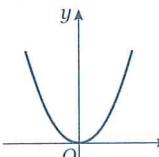
8 a $(-2, 3)$ 9 a $(0, 2)$ 

10 $(x - 2)^2 + (y - 3)^2 = 36$

11 centre $= (-2, 5)$, radius $= 4$ **Don't forget!**

* a straight line

* parabola

* 

* touches the curve but does not cross it

* perpendicular

* $x = 0$ * $y = 0$

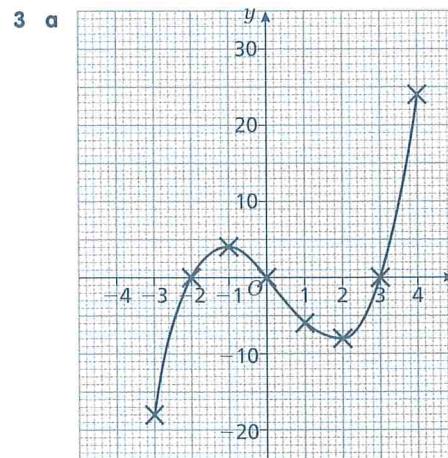
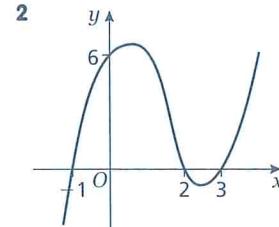
* the curve gets closer to but never touches or crosses

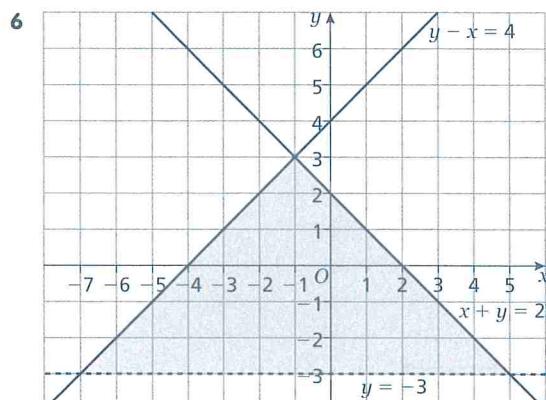
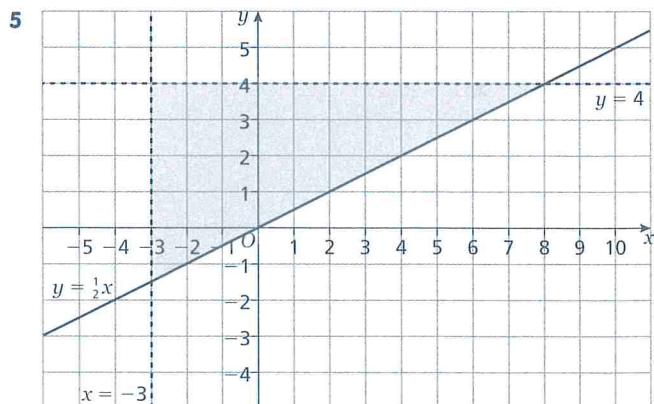
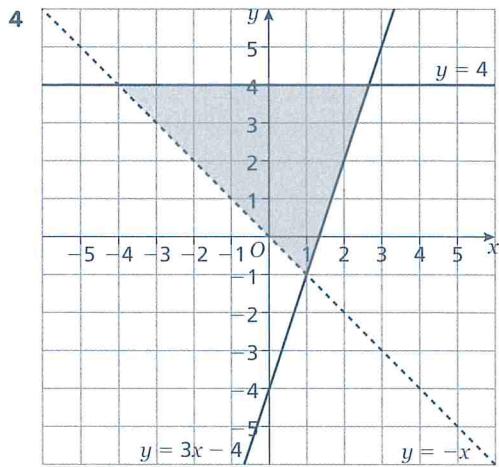
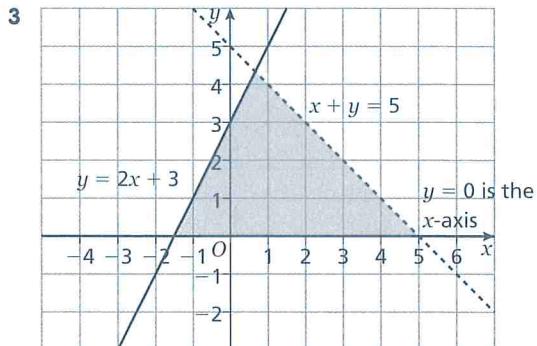
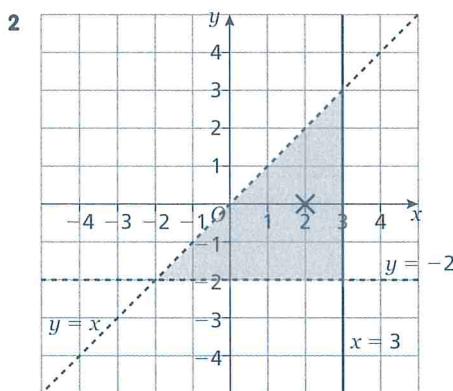
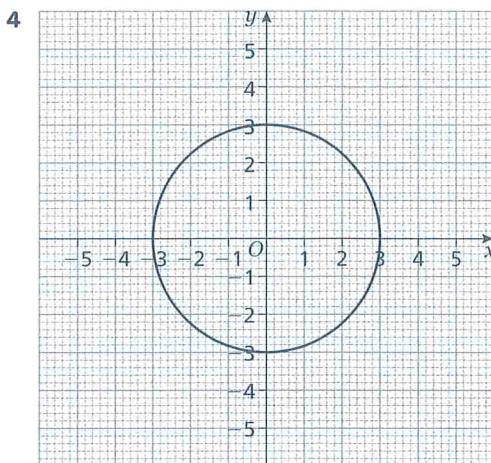
* complete the square

* turning points

* $(x - a)^2 + (y - b)^2 = r^2$; centre; radius; $x^2 + y^2 = r^2$ **Exam-style questions**

Equation	Graph
$y = 3^x$	D
$y = (x + 2)(x - 2)$	C
$y = (2 - x)(2 + x)$	A
$y = \frac{2}{x}$	E
$y = (x + 2)^2(1 - x)$	B

b $-2.4, 0.8, 2.6$



10 Inequalities

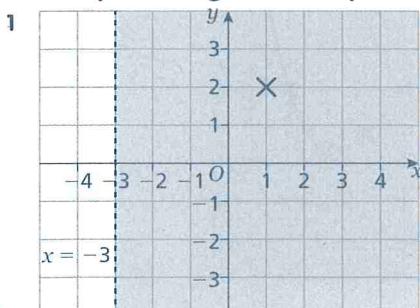
10.1 Solving linear inequalities

- 1 a $-2 \leq x < 4$ b $\frac{4}{5} < x \leq 2$
 c $2x < 12$ d $-5x \geq -10$
 e $4x - 8 > 27 - 3x$
 f $x > 5$
 g $x < -4$
 h $x < -3$
 i $x > 2$
 j $x \leq 1$
 k $x < -6$
 l $x < \frac{3}{2}$
 m $x > 5$ (which also satisfies $x \geq 3$)

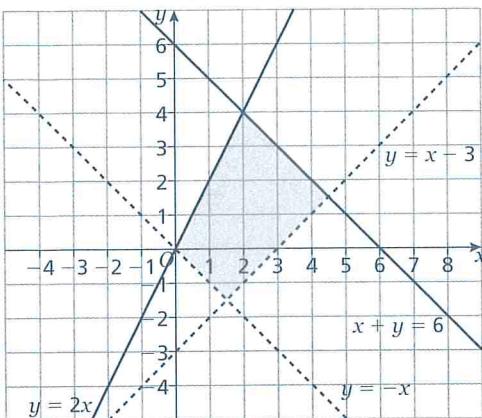
10.2 Solving quadratic inequalities

- 1 $(x + 3)(x + 2) = 0$
 x = -3, x = -2
 x < -3 or x > -2
- 2 $x(x - 5) = 0$
 x = 0, x = 5
-
- 3 $x^2 + 3x - 10 = 0$
 (x + 5)(x - 2) = 0
 x = -5, x = 2
-
- 4 $x \leq -2$ or $x \geq 6$
- 5 $-7 \leq x \leq 4$
- 6 $\frac{1}{2} < x < 3$
- 7 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$
- 8 $-3 \leq x \leq 4$
- 9 $2 < x < 2\frac{1}{2}$
- 10 $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$

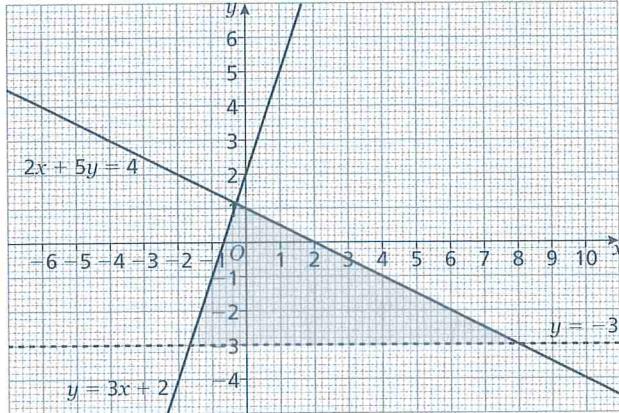
10.3 Representing linear inequalities on a graph



7



8

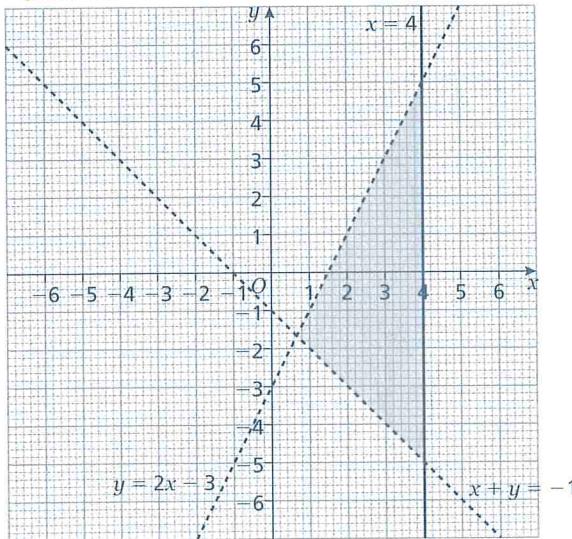
**Don't forget!**

- * a negative number
- * solve; sketch the graph; values
- * shading regions
- * unbroken (solid)
- * broken lines

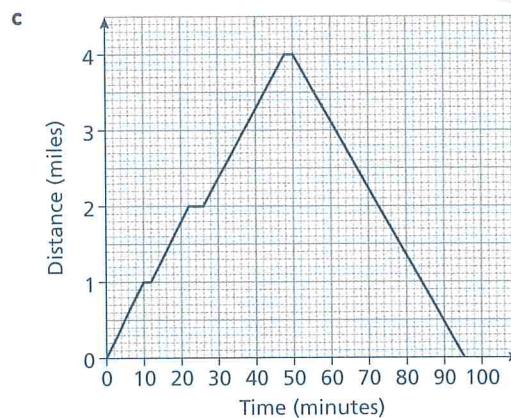
Exam-style questions

1 $-3 \leq x \leq 2$

2

**11 Distance-time and speed-time graphs****11.1 Distance-time graphs**

- 1 a 2 or 4 (depending on whether you've counted the start and finish)
 b $6 \div 10 = 0.6 \text{ m/s}$
 c $21 \div 20 = 1.05 \text{ m/s}$
 d Between 0 and 10 seconds
- 2 a 15 min
 b 15 miles
 c 6 mph
 d 48 mph
- 3 a 2 min
 b 10 min

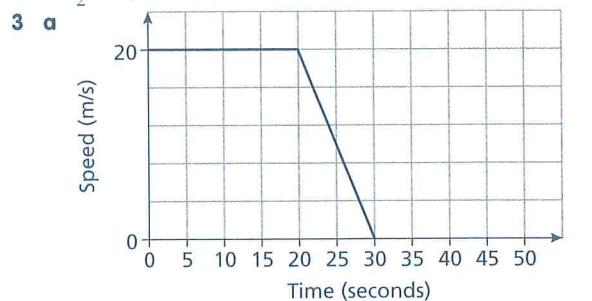


4 a C

b A

11.2 Speed-time graphs

- 1 a $9 \div 12 = 0.75 \text{ m/s}^2$
 b Method 1: $\frac{1}{2} \times 9 \times (20 + 32) = \frac{1}{2} \times 9 \times 52 = 234 \text{ m}$
 Method 2: $\frac{1}{2} \times 9 \times 12 + 9 \times 20 = 54 + 180 = 234 \text{ m}$
- 2 a $4 \div 8 = 0.5 \text{ m/s}^2$
 b $\frac{1}{2} \times 4(12 + 22) = 68 \text{ m}$



3 a 2 m/s

4 a 100 km/h^2

5 a 4 m/s^2

6 a 30 m/s

7 a 5 s

c 100 m

b 57 miles

b 150 m

c -2 m/s^2

b 1 m/s^2

b 275 m

Don't forget!

- * distance
- * the time taken
- * speed
- * faster
- * no movement
- * speed
- * the time taken to travel
- * constant speed
- * acceleration
- * deceleration
- * the distance travelled

Exam-style questions

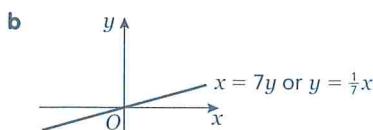
1 a 4 m/s^2

b 150 m

12 Direct and inverse proportion**12.1 Direct proportion**

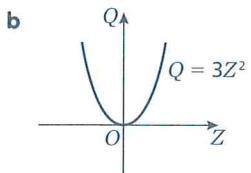
- 1 a $P \propto h$
 $P = kh$
 $56 = k \times 8$
 $k = 56 \div 8 = 7$
 $P = 7h$
- 2 a $y = kx^2$
 $45 = k \times 3^2$
 $k = 45 \div 9 = 5$
 $y = 5x^2$
- c $y = 5x^2$
 $20 = 5 \times x^2$
 $x^2 = 20 \div 5 = 4$
 $x = 2$
- b $P = 7h$
 $P = 7 \times 11$
 $P = £77$
- b $y = 5x^2$
 $y = 5 \times 5^2$
 $y = 125$

3 a $x = 7y$



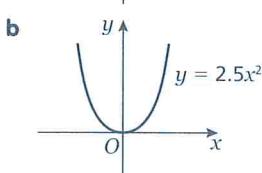
c 91
d 9

4 a $Q = 3Z^2$



c 75
d 10

5 a $y = 2.5x^2$



c 6

6 a $B = 2\sqrt{C}$

b 16

7 a $C = \frac{2}{3}D$

b 300

8 a $x = 3y$

b 11.1

9 a $m = 2n^3$

b 5

12.2 Inverse proportion

1 a $100 = \frac{k}{10}$

b $P = \frac{1000}{Q}$

$k = 1000 \times 10$

$20 = \frac{1000}{Q}$

$k = 1000$

$Q = 1000 \div 20$

$P = \frac{1000}{Q}$

$Q = 50$

2 a $y \propto \frac{1}{\sqrt{x}}$

b $y = \frac{5}{\sqrt{x}}$

$y = \frac{k}{\sqrt{x}}$

$5 = \frac{5}{\sqrt{x}}$

$1 = \frac{k}{\sqrt{25}}$

$\sqrt{x} = 5 \div 5 = 1$

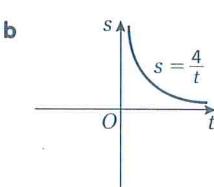
$k = 1 \times 5$

$x = 1$

$k = 5$

$y = \frac{5}{\sqrt{x}}$

3 a $s = \frac{4}{t}$

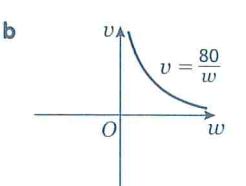


c 4

4 a $a = \frac{100}{b}$

b 2

5 a $v = \frac{80}{w}$



c 40

6 a $L = \frac{36}{W}$

b 6

7 a $s = \frac{72}{t}$

b 24

c 4

8 a $y = \frac{16}{x^2}$

b 1

9 a $a = \frac{0.2}{b}$

b 0.1

c 0.1

Don't forget!

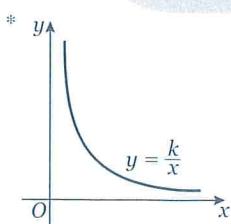
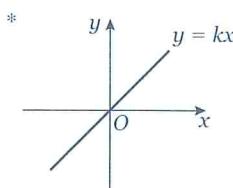
* direct

* inverse

* \propto

$\propto x; = kx$

$\propto \frac{1}{x}; = \frac{k}{x}$



Exam-style questions

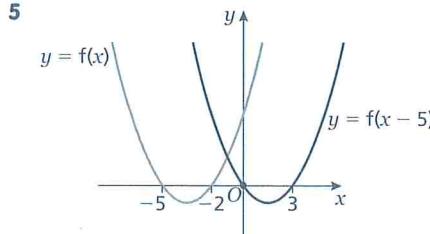
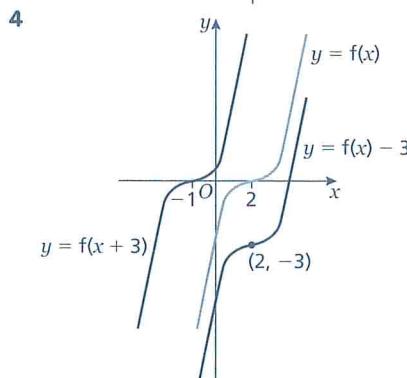
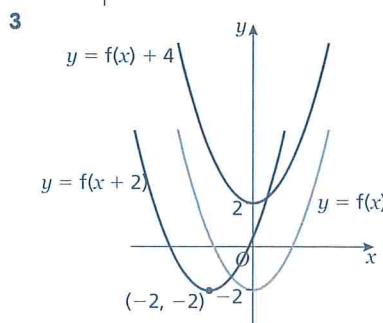
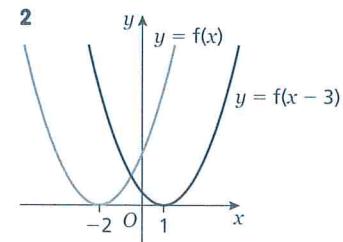
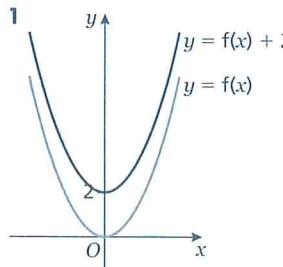
1 a $A = 3B^2$

b $\frac{3}{4}$

c 0.6

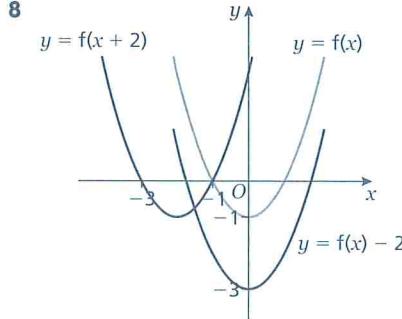
13 Transformations of functions

13.1 Applying the transformations $y = f(x) \pm a$ and $y = f(x \pm a)$ to the graph of $y = f(x)$

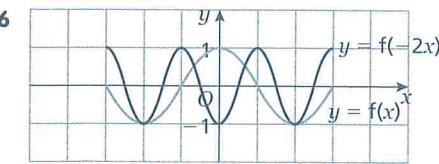
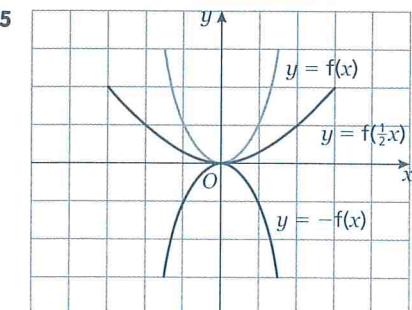
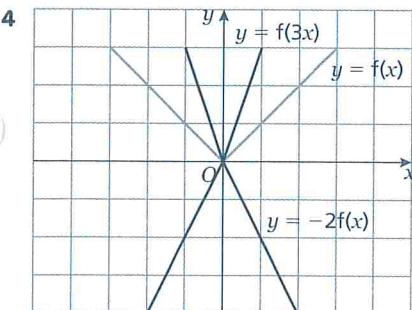
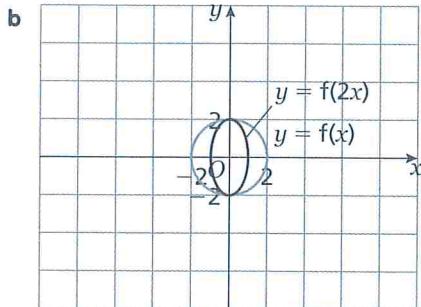
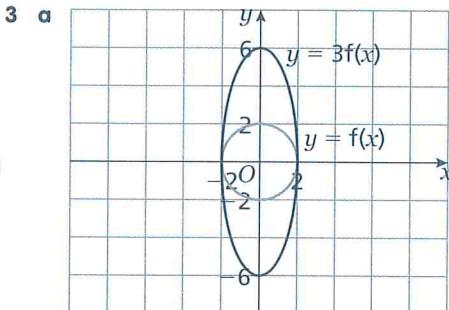
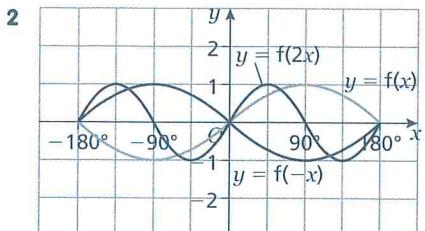
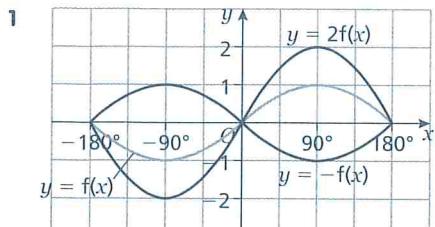


6 C₁: $y = f(x - 90^\circ)$
C₂: $y = f(x) - 2$

7 C₁: $y = f(x - 5)$
C₂: $y = f(x) - 3$



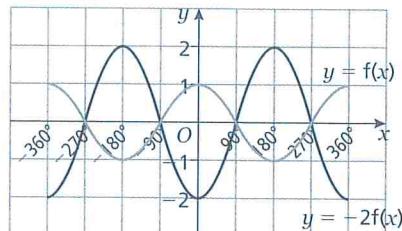
13.2 Applying the transformations $y = f(\pm ax)$ and $y = \pm af(x)$ to the graph of $y = f(x)$



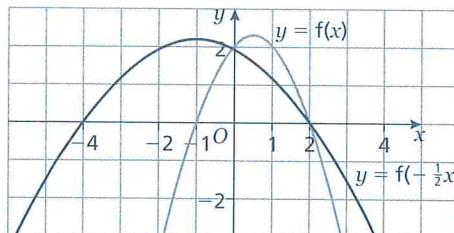
7 $y = f(2x)$

8 $y = -2f(2x)$ or $y = 2f(-2x)$

9 a, b



10 a, b

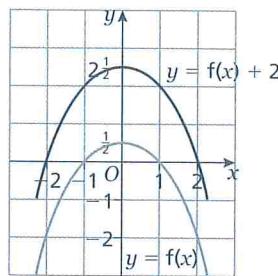


Don't forget!

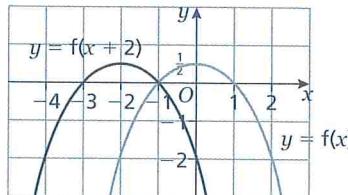
- * y
- * x ; left; right
- * $\frac{1}{a}$; x
- * $\frac{1}{a}$; x ; y
- * a ; y
- * a ; y ; x

Exam-style questions

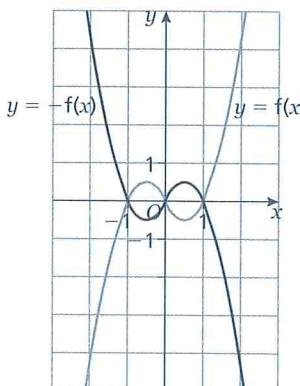
1 a



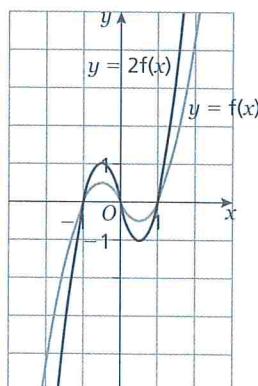
b



2 a



b



14 Area under a curve

14.1 The trapezium rule

1 $h = 1$

x	0	1	2	3
$y = (3 - x)(x + 2)$	6	6	4	0

$$y_0 = 6, y_1 = 6, y_2 = 4, y_3 = 0$$

$$A = \frac{1}{2} \times 1 \times [6 + 2(6 + 4) + 0]$$

$$= \frac{1}{2} [26]$$

$$= 13 \text{ sq units}$$

2 $h = \frac{10 - 4}{3} = 2$

x	4	6	8	10
y coordinate for the curve	7	12	13	4
y coordinate for the straight line	7	6	5	4

$y_0 = 0, y_1 = 6, y_2 = 8, y_3 = 0$

$A = \frac{1}{2} \times 2 [0 + 2(6 + 8) + 0]$

$= 1 \times 28$

= 28 sq units

3 34 sq units

6 $25\frac{1}{4}$ sq units

9 $26\frac{7}{8}$ sq units

4 149 sq units

7 35 sq units

10 56 sq units

5 14 sq units

8 42 sq units

11 $6\frac{1}{4}$ sq units

Don't forget!

* the area under a curve

* Area = $\frac{1}{2} h[y_0 + 2(y_1 + y_2 \dots + y_{n-1}) + y_n]$; the values of y for each value of x used

* number of equal strips the area has been divided up into; the vertical boundaries of the area

* the number of strips, n

* $= \frac{b-a}{n}$

Exam-style questions

1 71.25 sq units

2 35 sq units

3 72 sq units

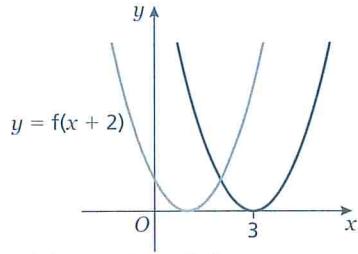
Practice Paper

1 $x = -4, y = -5$

3 $4y^2 - 6xy - 6x^2$

4 a $(2x - 3)(x + 1)$

5 a

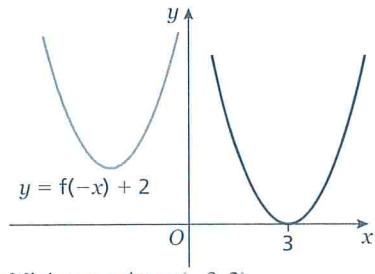


Minimum point at $(-2, 0)$

2 $m = \pm\sqrt{\frac{k}{6}}$

b 525

b



Minimum point at $(0, 2)$

6 a $3 \pm \sqrt{2}$

7 a $\frac{1}{2x}$

8 a $b = 1, c = -6$

9 a $x > \frac{5}{3}$

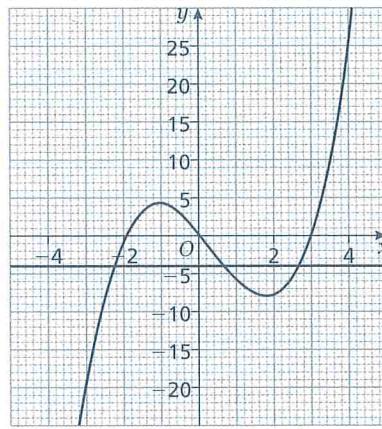
10 a 399

11 a $\frac{4-1}{6-2} \times \frac{8-4}{3-6} = -1$

12 a

x	-3	-2	-1	0	1	2	3	4
y	-18	0	4	0	-6	-8	0	24

b



c x is approximately $-2.3, 0.6$ or 2.7

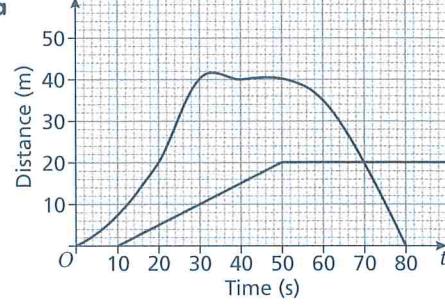
13 a $\frac{4x-3}{x^2-1}$

b $\frac{3}{4}$

14 a $16p^2 - 4 \times 4 \times (4p + 5) = 16p^2 - 64p - 80$

b $-1, 5$

15 a



b 70

16 a $(3, -2)$

c 1.7 ± 0.2 m/s

17 a 540 g

b $C \propto d$

$C = kd$

$36 = k \times 12$

$C = 3d$

$= 3\sqrt{\frac{3m}{5}}$

$= 3 \times \sqrt{\frac{9m}{15}}$

$= 9\sqrt{\frac{m}{15}}$

18 a $(4, 12)$

b $-2 \pm 2\sqrt{3}$

b 14 sq units