2 Data Representation and Interpretation

2.1 Data Tables

Worked Example 1

This timetable is part of a rail timetable for trains from the south west of England.

Saturdays							
Penzance	0715	—	—	—	—	0750	—	0846
Plymouth	0912	—	0935	—	—	1000	—	1035
Ivybridge	—	—	—	—	—	1014	—	—
Totnes	—	—	1002	—	—	1030	—	—
Paignton	—	0940	—	1001	1017	—	1030	—
Torquay	—	0946	—	1008	1022	—	1040	—
Torre	—	—	—	—	1025	—	—	—
Newton Abbot	0948	0958	1015	1020	1035	1044	1052	1112
Teignmouth	—	1005	1021	1026	1042	—	1101	—
Dawlish	—	1010	1026	1033	1047	—	1108	—
Dawlish Warren	—	—	—	1040	1051	—	1114	—
Starcross	—	—	—	—	1054	—	—	—
Exeter St. Thomas	—	—	—	—	1103	—	—	—
Exeter St. David's	1007	1023	1038	1052	1107	—	1127	1131
Tiverton Parkway	—	—	1055	—	—	—	—	1148
Taunton	1037	1054	1108	1118	—	—	1207	1201
Bristol Temple Meads	1115	1154	—	1158	—	—	1255	—
London Paddington	—	—	1325	—	—	—	—	1410

(a) Stewart wants to travel from Penzance to London Paddington. Describe the different options shown on this timetable.

(b) How can John get from Plymouth to Bristol Temple Meads?

Solution

(a) There are several possible options, including:

    (i) leave Penzance at 0715 and arrive at Newton Abbot at 0948. Then leave Newton Abbot at 1015 and arrive at London Paddington at 1325;

    (ii) leave Penzance at 0846 and travel direct to London Paddington, arriving at 1410.

(b) Possible options include:

    (i) leave Plymouth at 0912 and travel direct to Bristol Temple Meads, arriving at 1115;

    (ii) leave Plymouth at 0935 and travel to Newton Abbot arriving at 1015. Then leave Newton Abbot at 1020 and travel to Bristol Temple Meads, arriving at 1158;

2.1

(iii)   leave Plymouth at 1000 and travel to Newton Abbot, arriving at 1044. Then leave Newton Abbot at 1052 and travel to Bristol Temple Meads, arriving at 1255.

Worked Example 2

The chart can be used to find the mileage between some Scottish towns and cities.

Aberdeen						
115	Edinburgh					
152	130	Fort William				
142	44	103	Glasgow			
104	156	66	169	Inverness		
81	42	103	61	115	Perth	
226	123	183	84	249	145	Stranraer

Find the distances between:

(a)   Fort William and Perth,

(b)   Edinburgh and Stranraer.

(c)   Which two places are furthest apart?

Solution

(a)   To find the distance between Fort William and Perth look in the square where the two highlighted lines meet. So the distance is 103 miles.

Aberdeen						
115	Edinburgh					
152	130	Fort William				
142	44	103	Glasgow			
104	156	66	169	Inverness		
81	42	(103)	61	115	Perth	
226	123	183	84	249	145	Stranraer

2.1

(b)   Using the same approach for Edinburgh and Stranraer gives 123 miles.

	Aberdeen	Edinburgh	Fort William	Glasgow	Inverness	Perth
Edinburgh	115					
Fort William	152	130				
Glasgow	142	44	103			
Inverness	104	156	66	169		
Perth	81	42	103	61	115	
Stranraer	226	(123)	183	84	249	145

(c)   The largest number in the table is 249.

Using the highlight, this is the distance between Inverness and Stranraer. So these two places are furthest apart.

	Aberdeen	Edinburgh	Fort William	Glasgow	Inverness	Perth
Edinburgh	115					
Fort William	152	130				
Glasgow	142	44	103			
Inverness	104	156	66	169		
Perth	81	42	103	61	115	
Stranraer	226	123	183	84	(249)	145

2.1

Exercises

1. Mike lives in Paignton and works in Exeter, close to the Central Station. Use the following timetable to answer these questions.

    (a) He starts work at 9.00 am. Which train should he catch?

    (b) One day he misses his normal train. What is the earliest time he can get to Exeter Central?

    (c) Mike can walk from Exeter St. David's to his office in 15 minutes. What should he do if he misses his usual train?

    (d) Sometimes Mike has to go to Bristol or London. How can he get to these places as early as possible and at what time does he arrive?

Mondays to Fridays								
Penzance	—	—	—	—	—	—	0641	—
Plymouth	—	0725	0735	—	—	0818	0832	—
Ivybridge	—	—	—	—	—	0832	—	—
Totnes	—	0752	0802	—	—	0848	—	—
Paignton	0723	—	—	0800	0826	—	—	0856
Torquay	0728	—	—	0805	0832	—	—	0901
Torre	0731	—	—	0808	—	—	—	0904
Newton Abbot	0740	0805	0815	0818	0844	0900	0908	0914
Teignmouth	0747	—	—	0825	—	—	—	0921
Dawlish	0752	—	—	0830	—	—	—	0926
Dawlish Warren	0756	—	—	—	—	—	—	0929
Starcross	0759	—	—	0835	—	—	—	0933
Exeter St. Thomas	0808	—	—	0844	—	—	—	0942
Exeter St. David's	0812	0824	0834	0846	0903	—	0930	0945
Exeter Central	0818	0836	0853	0853	0923	—	0940	0953
Exmouth	0848	—	0918	0918	0948	—	—	1018
Barnstaple	—	—	—	0955	—	—	—	—
Tiverton Parkway	—	0841	0851	—	0920	—	—	—
Taunton	—	0854	0904	—	0933	—	0959	—
Bristol Temple Meads	—	0932	—	—	1009	—	1049	—
London Paddington	—	—	1110	—	—	—	—	—

2. Use the following timetable to answer these questions about James' journey.

    (a) James catches the 1927 at Reading. What time does this arrive at Cardiff Central?

    (b) He wanted to arrive at Cardiff before 11.00 pm. Could he have caught a later train?

    (c) What is the latest train he could have caught from Reading to arrive at Cardiff before 11.00 pm?

2.1

London → Bristol Parkway - Cardiff - Swansea Bath - Bristol Temple Meads

Mondays to Fridays continuation

London Paddington	**1815**	**1830**	**1845**	**1900**	**1915**	**2000**	**2015**	**2100**	**2115**
Reading	**1840**	**1857**	**1910**	**1927**	**1942**	**2027**	**2043**	**2127**	**2145**
Didcot Parkway	—	**1913**	**1925**	—	**1957**	—	**2058**	—	**2203**
Swindon	**1910**	**1934**	**1946**	**1959**	**2018**	**2059**	**2119**	**2200**	**2224**
Chippenham	**1923**	—	**1959**	—	**2032**	—	**2133**	—	**2238**
Bath Spa	**1938**	—	**2012**	—	**2044**	—	**2146**	—	**2251**
Bristol Parkway	—	**2002**	—	**2027**	—	**2127**	—	**2228**	—
Bristol Temple Meads	**1956**	—	**2025**	—	**2100**	—	**2201**	—	**2306**
Weston-super-Mare	**2023**	—	—	—	**2140**	—	**2235**	—	—
Newport	—	**2024**	—	**2049**	—	**2149**	—	**2250**	—
Cardiff Central	—	**2041**	—	**2106**	—	**2206**	—	**2312**	—
Bridgend	—	**2101**	—	**2126**	—	**2226**	—	**2332**	—
Port Talbot Parkway	—	**2112**	—	**2137**	—	**2237**	—	**2343**	—
Neath	—	**2120**	—	**2145**	—	**2245**	—	**2351**	—
Swansea	—	**2135**	—	**2200**	—	**2300**	—	**0005**	—

3.   Farharaz arrives by taxi at London Paddington at 0800. Use the timetable below to answer the following questions.

London - Reading - Bristol → Taunton Exeter - Torbay - Plymouth - Penzance

Mondays to Fridays

London Paddington	**0530**	**0740**	**0815**	**0845**	**0935**	**0945**	**1035**	**1135**	**1235**
Reading	**0610**	**0805**	**0845**	**0913**	**1003**	**1015**	**1103**	**1203**	**1303**
Newbury	—	—	—	—	**1018**	—	—	—	—
Pewsey	—	—	—	—	**1037**	—	—	—	—
Westbury	—	—	—	—	**1056**	—	—	**1248**	**1348**
Castle Cary	—	—	—	—	—	—	—	**1305**	—
Bristol Temple Meads	**0750**	**0915**	**1008**	**1028**	—	**1127**	—	**1328**	**1423**
Taunton	**0843**	**0952**	**1051**	**1108**	**1133**	—	—	**1328**	**1423**
Tiverton Parkway	**0857**	—	—	**1120**	**1145**	—	—	**1340**	—
Exeter St Davids	**0913**	**1016**	**1117**	**1135**	**1200**	**1235**	**1239**	**1355**	**1449**
Dawlish	**0928**	—	—	**1150**	—	—	—	—	—
Teignmouth	**0933**	—	—	**1155**	—	—	—	—	—
Newton Abbot	**0941**	**1040**	**1141**	**1203**	**1224**	—	—	**1419**	**1513**
Totnes	**0952**	**1052**	—	—	**1235**	—	—	**1433**	—
Plymouth	**1023**	**1120**	**1222**	**1240**	**1310**	—	**1339**	**1505**	**1551**
Liskeard	**1051**	**1149**	**1311**	—	—	—	**1404**	**1547**	**1619**
Bodmin Parkway	**1102**	**1201**	**1323**	—	—	—	**1416**	**1559**	**1631**
Par	**1114**	**1211**	**1334**	—	—	—	**1427**	**1611**	**1644**
St Austell	**1121**	**1219**	**1341**	—	—	—	**1435**	**1618**	**1652**
Truro	**1138**	**1237**	**1359**	—	—	—	**1453**	**1636**	**1710**
Redruth	**1150**	**1249**	**1411**	—	—	—	**1505**	**1649**	**1722**
Camborne	**1156**	**1256**	**1417**	—	—	—	**1512**	**1656**	**1729**
St Erth	**1207**	**1307**	**1428**	—	—	—	**1523**	—	**1741**
Penzance	**1217**	**1320**	**1439**	—	—	—	**1535**	**1716**	**1755**

(a)   Can he get to Totnes by train to meet his friend Lucy at noon?

(b)   What is the earliest time he can arrive in Totnes?

(c)   Lucy can meet Farharaz at Newton Abbot. What is the earliest time he can be there?

2.1

4. The table gives the distances, in miles, between some towns and cities.

   (a) Find the distances between:

      (i) Birmingham and Leeds

      (ii) Manchester and Bradford

      (iii) Sheffield and Coventry.

   (b) Nargis travels from Birmingham to Leeds and then to Manchester.

      (i) How far does he travel?

      (ii) How would the distance he travels be reduced if he went to Manchester and then to Leeds?

Birmingham	Bradford	Coventry	Derby	Doncaster	Leeds	Manchester	Sheffield
108							
18	114						
40	74	40					
93	34	92	54				
109	9	110	70	28			
80	34	94	59	51	40		
77	37	78	37	18	33	38	

   (c) Denise is going to Birmingham from Leeds. She must stop in either Bradford or Sheffield on the way. Which is the shorter route?

5. The table gives the distances in km between 3 ports, Calais, St. Malo and Boulogne, and some holiday destinations in Europe.

Calais	St. Malo	Boulogne	Florence	Interlaken	Venice	Quimper	Ile de Re	Biarritz
1422	1560	1430						
811	1169	819	612					
1370	1561	1388	307	563				
729	244	707	1708	1172	1710			
704	344	670	1494	995	1355	400		
1110	729	1152	1358	1233	1504	787	409	

   (a) The Eccles family decide to go to Biarritz for their holiday. Which of the three ports (Calais, St. Malo or Boulogne) is closest to Biarritz?

   (b) They decide to start their holiday at Calais, and also to visit Venice before they return to Calais. How far do they have to travel in total?

   (c) Their friends, the Morse family, decide to travel from St. Malo to Biarritz, then Interlaken and return to Boulogne. How far do they have to travel?

   (d) Which holiday destination is closest to St. Malo and which is closest to Calais?

2.1

6. In a school 30 students took GCSE exams in both Maths and Physics. Their results are given in this table.

Maths Grade

		A	B	C	D	E
	A	2	3			
	B	1	1	4		
Physics Grade	C		2	3	2	
	D			4	2	2
	E		1	0	2	1

(a) How many students got the same grade in both subjects?

(b) How many students got a higher grade in Physics than in Maths?

(c) Which was the most common grade in Physics?

7. The table shows the sports options selected by a group of students in each of their years in secondary school. In each year, each student chose just one sport.

	Hockey	Football	Tennis	Swimming
Year 7	6	18	5	14
Year 8	5	16	7	15
Year 9	7	14	10	12
Year 10	2	12	10	19
Year 11	8	13	12	10

(a) How many chose tennis in Year 8?

(b) How many more chose football in Year 7 than in Year 10?

(c) In which years was football the most popular sport?

(d) (i) In which year was swimming the most popular sport?

(ii) How many students were there in this group?

2.1

8. The table below shows the cost of a week at a large holiday centre. The cost varies according to the number of people in the party and the type of accommodation booked. There is a £20 reduction for each child.

Accommodation Type

	Saver	*Comfortable*	*Luxury*
4	£180	£260	£368
5	£220	£320	£454
6	£265	£385	£541
7	£305	£449	£630
8	£340	£507	£704

Number of people

(a) How much would it cost for Mr and Mrs Jones and their 4 children to stay in 'Comfortable' accommodation for one week?

(b) How much more would it cost if they booked 'Luxury' accommodation?

(c) How much would they save in 'Saver' accommodation compared with 'Luxury' accommodation?

(d) (i) How much would it cost if two grandparents came with the family and they all stayed in 'Saver' accommodation?

   (ii) By how much does this differ from the total in part (a)?

9. In a privatised railway company, there are 84 male conductors and 56 female conductors. Conductors can either be senior or standard. There is a total of 28 senior conductors and there are 48 female standard conductors.

(a) Copy and complete the two-way table to show the number of male and female conductors who are senior or standard.

	Male	*Female*
Standard		
Senior		

(b) Comment on the results.

10. Each student in a class chose *one* sport. The numbers of choices were put in a table.

		Outdoor Sports		**Indoor Sports**	
		Hockey	Tennis	Badminton	Squash
Year 11	Girls	12	10	15	5
	Boys	10	15	7	19
Year 10	Girls	14	9	17	3
	Boys	15	12	11	13

2.1

(a)   How many students chose hockey?

(b)   How many more girls chose tennis than squash?

(c)   One girl says that boys usually prefer outdoor sports. Do the figures in the
table support this view? Explain your answer.

(SEG)

11.   The two-way table shows the number of students achieving grades **A** to **E** in
examinations in English and French.

English grade

	E	D	C	B	A
A			1	1	2
B		1		5	2
C		1	7	2	
D		2	3	1	
E	3	1			

French grade

(a)   How many of the students who achieved grade **B** in English achieved a
different grade in French?

(b)   How many students achieved the same grade in both subjects.

(c)   What does the table suggest about the grades achieved in English and
French?

(SEG)

12.   St. Margaret's School entered Y11 and Y10 pupils for the AQA mathematics
examination at the Foundation, Intermediate and Higher tiers.

(a)   How many Y10 pupils have entered for the examination?

(b)   (i)   What was the total number of pupils entered for the Intermediate tier?

       (ii)   What percentage of the pupils entered for the Intermediate tier were
              Y10 pupils?

		Foundation	Intermediate	Higher
Y11	Boys	25	18	7
	Girls	10	45	11
Y10	Boys	0	13	3
	Girls	0	14	1

2.2 | Stem and Leaf Plots

There are many ways of representing data. For example, you should already be familiar with

pictograms and *pie charts*

but there is another very simple way which quickly gives an overall view of the general characteristics of the data. This is called a

stem and leaf plot

and the following example illustrates how it works.

Suppose the marks gained out of 50 by 15 pupils in a Biology test are as given below.

27	36	24	17	35	18	23	25
34	25	41	18	22	24	42	

We form a *stem and leaf plot* by recording the marks with the 'tens' as the stem and the 'units' as the leaf, as shown opposite.

Stem	Leaf						
0							
1	7	8	8				
2	7	4	3	5	5	2	4
3	6	5	4				
4	1	2					

The leaf part is then reordered to give a final plot as shown.

This gives at a glance both an impression of the *spread of* the numbers and an indication of the *average*.

Stem	Leaf						
0							
1	7	8	8				
2	2	3	4	4	5	5	7
3	4	5	6				
4	1	2					

Worked Example 1

Form a stem and leaf plot for the following data.

21	7	9	22	17	15	31	5	17	22	19	18	23
10	17	18	21	5	9	16	22	17	19	21	20	

Solution

Without reordering we have,

Stem	Leaf									
0	7	9	5	5	9					
1	7	5	7	9	8	0	7	8	6	7	9
2	1	2	2	3	1	2	1	0		
3	1									

and reordered,

Stem	Leaf									
0	5	5	7	9	9					
1	0	5	6	7	7	7	7	8	8	9	9
2	0	1	1	1	2	2	2	3		
3	1									

2.2

Worked Example 2

Blood samples were taken from forty blood donors and the lead concentration (in mg per 100 ml) in each sample was determined. The results are given below.

39	24	19	31	65	53	25	17	30	28
25	30	60	22	31	44	24	38	18	18
36	64	43	25	23	28	20	42	30	45
25	24	41	45	35	32	28	53	17	28

(a)   Construct a stem and leaf diagram to represent these data.

(b)   For these data, write down the values of

    (i)   the range,

    (ii)   the median.

(c)   Describe what the distribution shows.

(NEAB)

Solution

(a)   Reading from the table,

Stem	Leaf												
0													
1	9	7	8	8	7								
2	4	5	8	5	2	4	5	3	8	0	5	4	8	8
3	9	1	0	0	1	8	6	0	5	2			
4	4	3	2	5	1	5							
5	3	3											
6	5	0	4										

and, reordering,

Stem	Leaf												
0													
1	7	7	8	8	9								
2	0	2	3	4	4	4	5	5	5	5	8	8	8	8
3	0	0	0	1	1	2	5	6	8	9			
4	1	2	3	4	5	5							
5	3	3											
6	0	4	5										

(b)   (i)   The range is $65 - 17 = 48$ mg per 100 ml.

    (ii)   Using these 40 data values, the median is the mean of the 20th and 21st values. This is $\frac{1}{2}(30 + 30) = 30$ mg per 100 ml.

(c)   The distribution shows that most people have fairly low levels of lead concentration in their blood.

2.2

Exercises

1. A class of 25 students obtained the following marks in a Mathematics test.

26	18	37	42	29
49	21	52	31	32
15	28	24	35	36
51	31	24	46	41
38	40	16	22	57

(a) Construct a stem and leaf diagram. Place the figures on the leaves in order of size.

(b) Using your stem and leaf diagram, or otherwise, find

   (i) the range,

   (ii) the median.

(NEAB)

2. The ages of drivers involved in fatal accidents in England during one week are given below.

17	82	40	48	21	35	23	24	18	57	62	45
20	21	33	27	24	37	58	69	65	19	15	21
28	71	43	31	73	26	18	21	34	35	51	63
23	65	22	45	23	27	18	19	32	25	61	36

(a) Illustrate this data using a stem and leaf plot.

(b) Describe the main features of the data.

3. The lengths, in seconds, of the tracks on a double album are:

Volume 1	203	288	249	215	254	283	266
	202	237	221	262	240	253	266
	246	273	203				
Volume 2	170	185	240	195	202	174	179
	182	195	263	190	210	183	201
	179						

2.2

(a)   Collect these data on a back-to-back stem and leaf diagram as started below. Use a second diagram to reorder the data.

Volume 1			Volume 2
		17	0
		18	5
		19	
	3	20	
	5	21	
		22	
		23	
	9	24	
	4	25	
		26	
		27	
3	8	28	

(b)   Use your back-to-back stem and leaf diagram to compare the length of tracks on volume 1 and volume 2.

(SEG)

2.3 Pictograms and Bar Charts

Pictograms and bar charts can be used for displaying data when the data are in discrete categories.

For example, the pictogram and bar chart below show the favourite sports of 25 children.

Pictogram

Football ♀ ♀ ♀ ♀ ♀ ♀ ♀ ♀
Tennis ♀ ♀ ♀ ♀ ♀
Swimming ♀ ♀ ♀ ♀ ♀ ♀
Cricket ♀ ♀
Snooker ♀ ♀ ♀ ♀

♀ Represents 1 child

Bar Chart

Number of Children

Sports
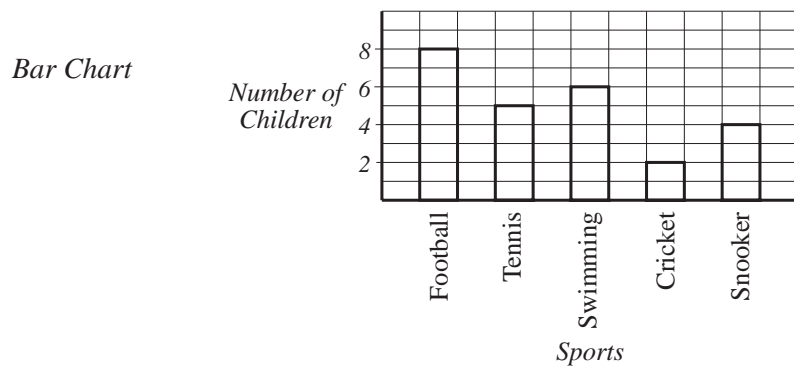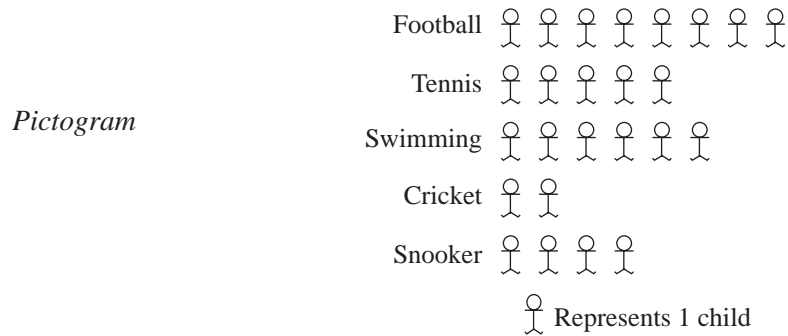
Worked Example 1

The pictogram shows the number of customers using a coffee shop during one week.

(a)   How many customers used the shop on Wednesday?

(b)   How many customers used the shop on Monday?

(c)   How many customers visited the coffee shop during the week?

Monday
Tuesday
Wednesday
Thursday
Friday
Saturday

Represents 20 customers
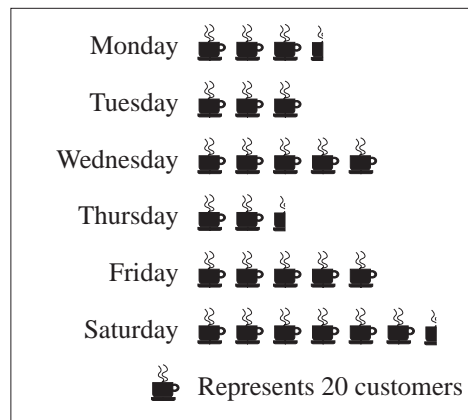
Solution

(a)   For Wednesday there are 5 symbols and each symbol represents 20 customers, so the number of customers was

$$5 \times 20 = 100$$

(b)   For Monday there are $3\frac{1}{2}$ symbols, so the number of customers was

$$3\frac{1}{2} \times 20 = 70$$

(c)   The total for the week is given by

$$70 + 60 + 100 + 50 + 100 + 130 = 510 \text{ customers.}$$

2.3

Worked Example 2

John asked each person in the class what their shoe size was. He obtained these results.

7	5	6	8	4	$5\frac{1}{2}$
$6\frac{1}{2}$	7	8	$7\frac{1}{2}$	$5\frac{1}{2}$	6
$6\frac{1}{2}$	$5\frac{1}{2}$	7	6	$6\frac{1}{2}$	8
7	5	$6\frac{1}{2}$	6	$7\frac{1}{2}$	7
$5\frac{1}{2}$	6	5	$5\frac{1}{2}$	6	$7\frac{1}{2}$

Draw a bar chart to show this data.

Solution

First the data can be entered into a tally chart.

Shoe Size		Total
4			1
$4\frac{1}{2}$		0
5					3
$5\frac{1}{2}$	⊦⊦⊦⊦	5
6	⊦⊦⊦⊦		6
$6\frac{1}{2}$	||||	4
7	⊦⊦⊦⊦	5
$7\frac{1}{2}$	|||	3
8					3
		30

The bar chart can be drawn as shown below.

Number of children (y-axis), *Shoe size* (x-axis)
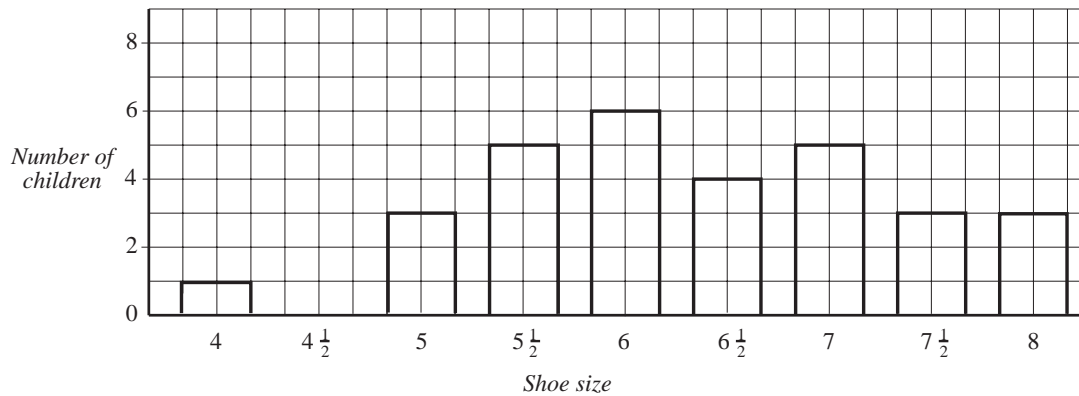
2.3

Exercises

1. Jenny kept a record of the average number of cans she recycled each week over a number of years. The pictogram shows her results.

    (a) In which year did she recycle most cans?

    (b) In which year did she recycle an average of 11 cans per week?

    (c) How many cans did she recycle each week in:

    (i) 1999,

    (ii) 1997,

    (iii) 2000,

    (iv) 2003 ? Give a reason for this estimate.

1996	♲ ♲
1997	♲ ♲ ♲ (
1998	♲ ♲ ♲ ♲ (
1999	♲ ♲ ♲ ♲ ♲
2000	♲ ♲ ♲ ♲ ♲ ♲
2001	♲ ♲ ♲ ♲ ♲ (
2002	♲ ♲ ♲ ♲ ♲ ♲ (

♲ Represents 2 cans
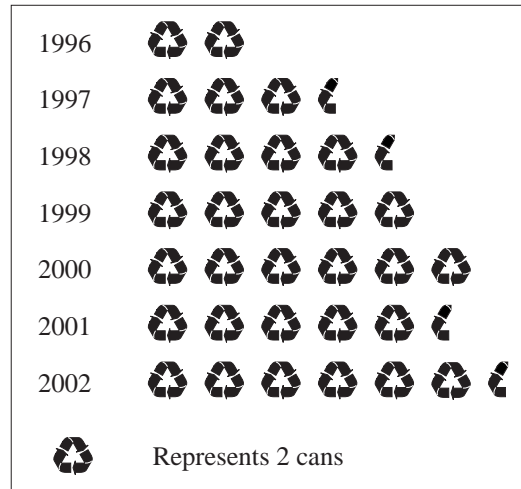
2. The pictogram shows how many suitcases were sold by a shop from 1996 to 2002, with one row missing.

1996	💼 💼 💼
1997	💼 💼 💼 💼
1998	💼 💼 💼
1999	💼 💼 💼 💼 💼
2000	💼 💼 💼 💼 💼 💼 💼
2001	
2002	💼 💼 💼 💼 💼 💼 💼

💼 Represents 100 suitcases
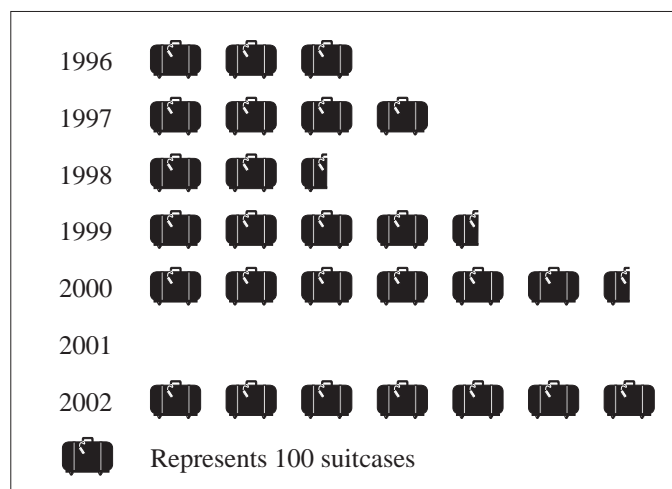
    (a) How many suitcases were sold in 1997?

    (b) What is the smallest number of suitcases sold in a year?

    (c) What is the greatest number of suitcases sold in a year?

    (d) In 2001 a total of 550 suitcases were sold. How many suitcases should appear in the missing row?

    (e) How many suitcases have been sold altogether?

    (f) Estimate how many suitcases might be sold in 2003.

3.   A class conducted a survey to find their favourite ice creams. The results were:

Favourite Ice Cream	Number of Children
Solero	9
Magnum	12
Mars	7
Feast	4

(a)   Draw a pictogram to show these results.

(b)   Represent this information in a bar chart.

(c)   What are the advantages of each type of representation of the data?

4.   A group of students recorded the number of vehicles passing their school in one hour. The results are recorded below.

Vehicle Type	Number of Vehicles
Cars	20
Vans	8
Lorries	3
Motorbikes	5
Buses	2

Represent this information with a bar chart and comment on the data.

5.   Draw a bar chart to show the data given in the table about the hours of sunshine per day at a number of resorts.

Resort	Hours of Sunshine per Day
Algarve	6
Benidorm	6
Eilat	7
Majorca	5
Mombasa	9
Tenerife	6
Torremolinos	6

2.3

6. The children on a school bus were asked which year group they were in. Their replies were:

10	7	7	10	11	9	8	7	8	9
7	9	11	11	8	8	9	7	10	10
11	8	9	7	10	11	11	11	11	7
7	7	8	7	8	9	10	10	9	8

Draw a bar chart to show this data.

7. A headteacher asked a class of Year 7 students how many younger brothers and sisters each student had. The results were:

0	1	2	1	0	0	1	2	1	1
2	0	0	1	1	2	3	4	1	1
2	1	2	0	0	3	2	1	5	1

(a) Draw a bar chart to illustrate this data.

(b) Why do you think the headteacher wanted this information?

8. A group of households were asked to give information about access to satellite, cable and terrestrial TV. The results are summarised below.
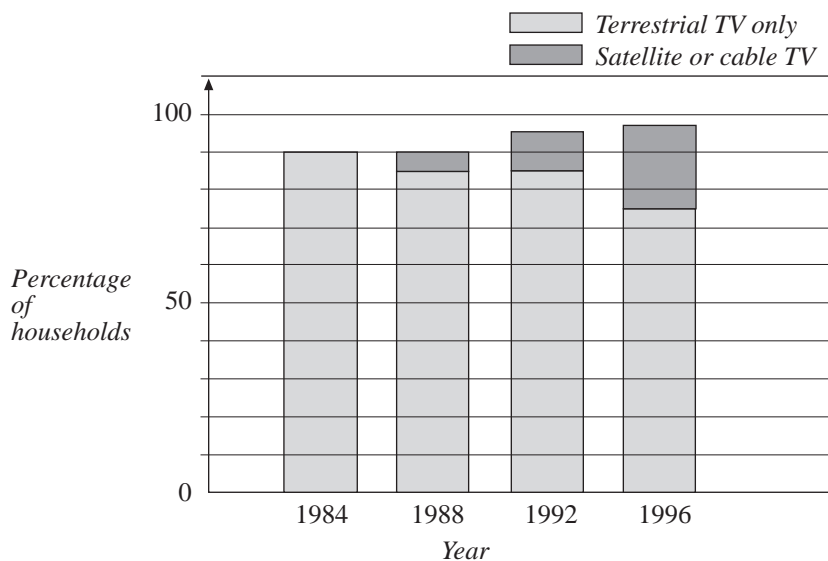
Use this presentation of the results to answer the following questions.

(a) What percentage of households had some form of TV in

    (i)   1990   (ii)   2002?

(b) What percentage of households had satellite or cable TV in

    (i)   1990   (ii)   2002?

(c) Comment on the trends in the results from the survey.

2.3

9.

1992	
1993	

 = 10 ships
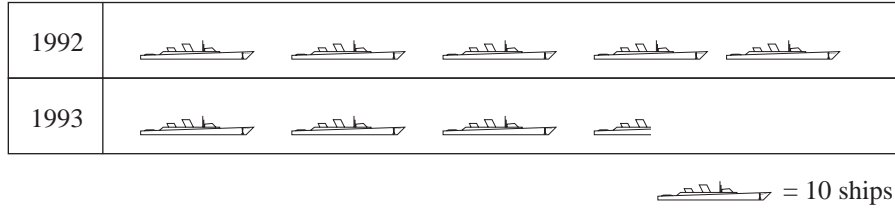
The diagram shows how many ships were in Mathsland's navy in 1992 and 1993.

Use the diagram to answer these questions.

(a)   How many ships were there in Mathsland's navy in 1992?

(b)   How many ships were there in Mathsland's navy in 1993?

(LON)

10.   The bar chart below shows the shoe sizes of a group of 16 year old boys.
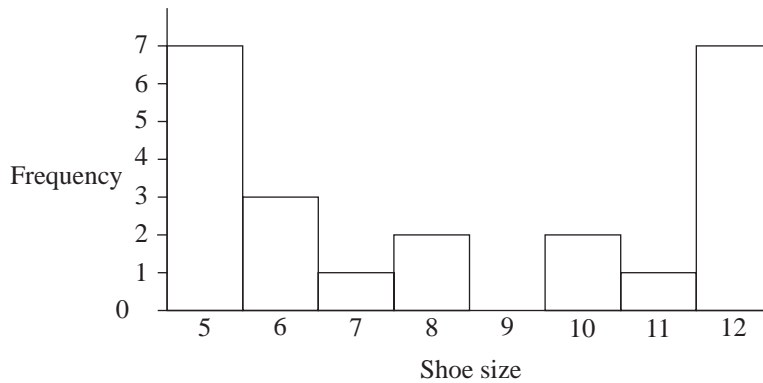
(a)   How many boys are there in the group?

(b)   Comment on the shape of the bar chart, saying whether or not this is the shape you would expect.

(MEG)

11.

COST OF UNEMPLOYMENT PER HEAD OF POPULATION

| SWEDEN £444 | GERMANY | UK £280 | IRELAND £268 | FRANCE £267 | USA £86 | ITALY £28 |

ACKNOWLEDGEMENT: *Today*, 29 September 1993
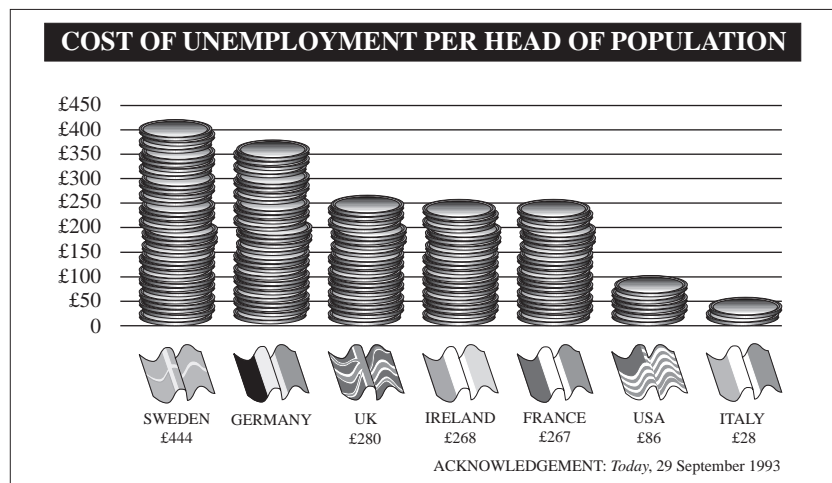
The pictogram represents, approximately, the cost of unemployment per head of population in seven developed countries. The exact figures for six of them are shown under their names.

2.3

(a)   There are 19 coins in the pile for Ireland. To the nearest whole number, how many pounds does one coin represent?

(b)   Estimate the cost of unemployment per head of population in Germany.

(c)   The cost of unemployment per head of population in Japan is £218.

How many coins would there be in the pile for Japan?

(d)   Explain why the pictogram is only approximate.

12.   Sally did a survey of car colours.

The notebook shows all her results.

w	w	r	w	g
r	w	r	b	r
r	r	w	r	r
b	w	r	r	w
w	g	r	w	r
g	w	w	b	r

Key:

w   white
b   blue
r   red
g   green

(a)   Copy and complete the frequency table.

COLOUR	TALLY	FREQUENCY
White		
Blue		
Red		
Green		

(b)   Show this information as a bar chart of the form shown here.

(SEG)

43
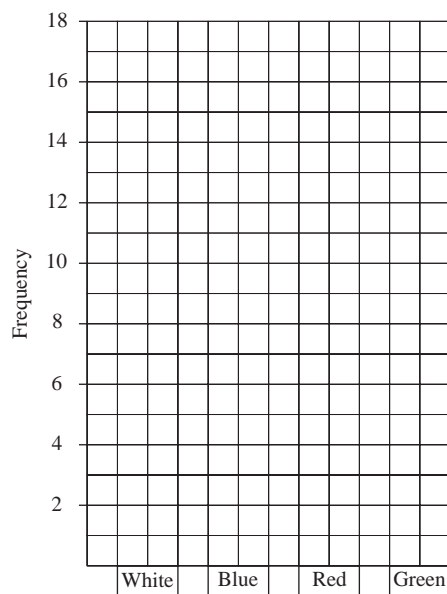
2.4 Pie Charts

Pie charts, which represent quantities as sectors of a circle, can be used to illustrate data. They are particularly effective if there is only a small number of items to illustrate. In total a complete circle, i.e. 360°, must always be used.

Worked Example 1

Tracey uses her pocket money of £18 per month in the following way.

Magazines	£4
Sweets	£3
Swimming	£6
Bus fares	£4
Money box	£1

Draw a pie chart to show how Tracey uses her pocket money.

Solution

Tracey has a total of £18.

$$\frac{360°}{18} = 20°$$

So 20° should be used for each £1. The angles needed are given in this list.

Magazines	$4 \times 20° = 80°$
Sweets	$3 \times 20° = 60°$
Swimming	$6 \times 20° = 120°$
Bus fares	$4 \times 20° = 80°$
Money box	$1 \times 20° = 20°$

The pie chart can now be drawn.

The diagram below shows the first section for magazines.

This is the completed pie chart.
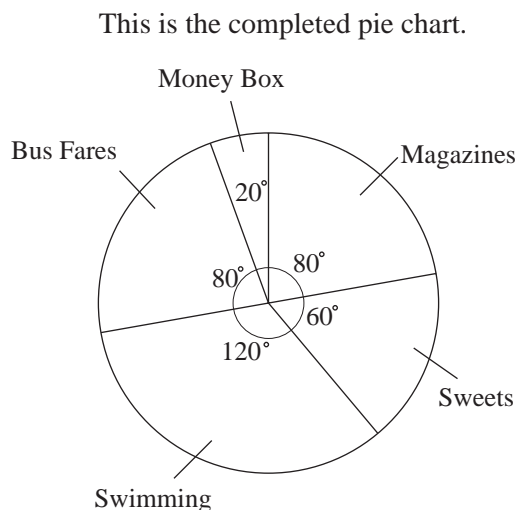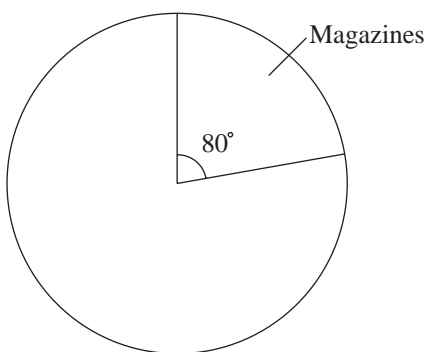
2.4

Worked Example 2

This pie chart was constructed after asking 72 children how they travel to school.

(a) How many children travel to school by:

(i) car, (ii) bike, (iii) bus?

(b) What percentage walk to school?
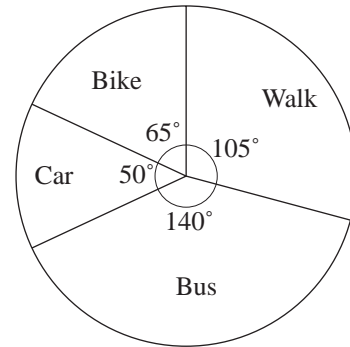
Solution

(a) There are 72 children so

$$\frac{360°}{72} = 5° \text{ per child.}$$

(i) The angle for travelling by car is $50°$ so $\dfrac{50°}{5°} = 10$ children travel by car.

(ii) The angle for travelling by bike is $65°$ so $\dfrac{65°}{5°} = 13$ children travel by bike.

(iii) The angle for travelling by bus is $140°$ so $\dfrac{140°}{5°} = 28$ children travel by bus.

(b) The number of children who walk to school is given by

$$\frac{105°}{5°} = 21 \text{ children}$$

so the percentage of children who walk is

$$\frac{21}{72} \times 100 \approx 29.2\%$$

Note

Alternatively, you could just use the angles in the pie chart to give

$$\frac{105°}{360°} \times 100 \approx 29.2\%$$

Comparative Pie Charts

If you want to use pie charts to make comparisons, then care must be taken that the areas of the circles are in proportion to the frequencies that they represent.

2.4

Exercises

1. In an opinion poll 360 people were asked who they would vote for in the next election. Their responses are:

Labour	150
Liberal Democrat	60
Conservative	100
Other	50

   Draw a pie chart to show this.

2. (a) Sarah recorded how she spent the last 24 hours. Her results are below.

Sleeping	9 hours
School	7 hours
Homework	2 hours
Watching TV	3 hours
Eating	1 hour
Travelling	2 hours

   Draw a pie chart to show this information.

   (b) Consider how you spend a day and comment on the differences between your day and Sarah's.

3. Emma has 720 stamps in her stamp collection. She has sorted them into three groups.

UK	*400 stamps*
Europe	*200 stamps,*
Other countries	*120 stamps*

   Draw a pie chart to show this information.

4. The 30 pupils in a class state their favourite sport. Their results are listed below.

Snooker	3
Football	9
Netball	6
Squash	2
Tennis	10

   Draw a pie chart to show this information.

2.4

5.   The pie chart shows how the time Ron spends watching television is split between different channels, for one day.
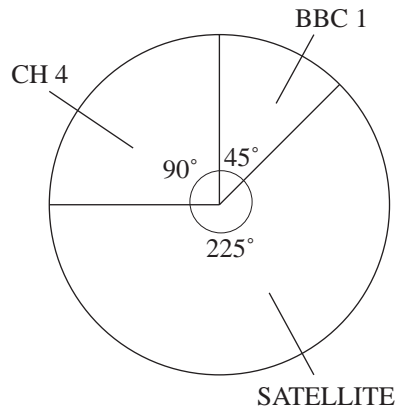
Ron spends 1 hour watching BBC1.

(a)   How long does he spend watching CH4?

(b)   How long does he spend watching satellite TV?

6.   Ahmed was given £60 on his birthday. The pie chart shows how he spent this money.
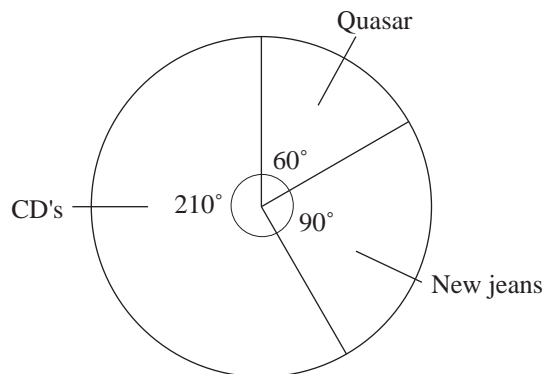
How much did he spend on:

(a)   Quasar,   (b)   his new jeans,   (c)   CDs?

7.   The pie chart shows the football teams supported by a class of children. There are 3 children who support Spurs.
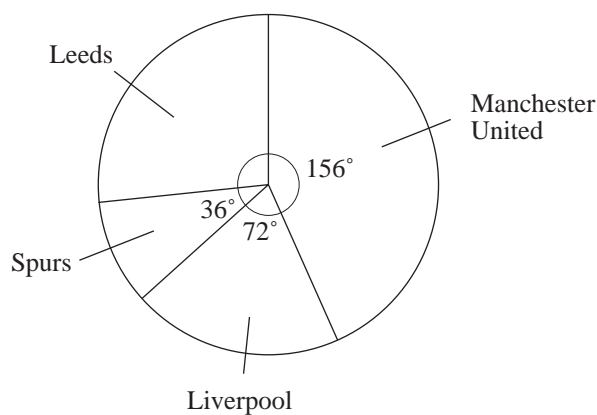
2.4

(a) What is the angle representing Leeds?

(b) How many children support Liverpool?

(c) How many children are there in the class?

(d) How many children support Manchester United?

8. A postman collects 720 letters from the letter boxes in a small town. They are sorted into First Class, Second Class and Air Mail. The pie chart shows the different numbers of each type.

How many letters of each type did he collect?

Second Class 175°   160° First Class

25°

Air Mail
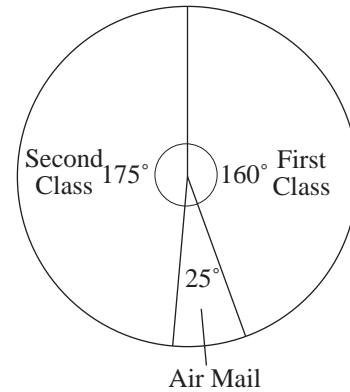
9. Sita spent £90.
The table shows what she spent it on.

Items	Amount spent
Bus fares	£12
Going out	£25
Clothes	£30
Records	£15
Others	£8
Total Spending	£90

Sita is asked to construct a pie chart to show her spending.

(a) Work out the angle of each sector in the pie chart.

Items	Angle of sector
Bus fares	
Going out	
Clothes	
Records	
Others	
Total of angles	360°

(b) Construct the pie chart to illustrate the data.

(c) What fraction of Sita's spending was on clothes?   *(LON)*

2.4

10.   Arthur spends £180 per week.

Item	Spending £
Food	30
Heating and lighting	12
Clothes	20
Other items	20
Housing	98
Total spending	180

The way in which he spends his money is shown in the table.

(a)   Draw a pie chart to show how Arthur spends his money.

This pie chart shows how the average person spends money.
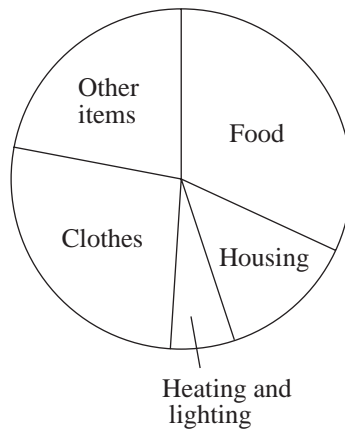
(b)   Describe **one** way in which Arthur's spending differs from the average
        person's spending.   *(SEG)*

11.   This question is about the way water is used in two Mozambique villages.

(a)   In village *A*, 324 litres of water are used each day.

        The pie chart shows how the water is used.
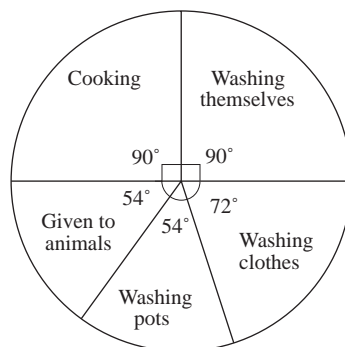
2.4

    (i)   How much water (in litres) is used
              each day for cooking?

    (ii)   What fraction of the water used is
              given to animals?

(b)   In village *B*, the water is used as follows:

Cooking	20%
Washing themselves	50%
Washing clothes	20%
Washing pots	10%

Represent this information in a pie chart.   *(MEG)*

12.   (a)   (i)   500 000 cars were stolen in England and Wales in 1990.

          Write the number 500 000 in words.

    (ii)   40% of the cars stolen in 1990 were never found.

          Calculate the number of cars that were never found.

(b)   The chart shows information about the people who stole cars in 1990.

Use the information in the diagram to answer these questions about car theft in 1990.
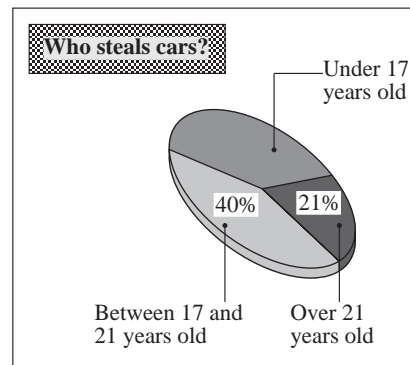
    (i)   What percentage of the cars stolen were taken by people aged under 17?

    (ii)   Which age group was the least likely to be involved in stealing cars?

    (iii)   What is the probability that a stolen car was taken by a person who was aged 17 or over?

          Give your answer as a decimal or a fraction.

(NEAB)

2.5 | Line Graphs

A *line graph* is drawn by plotting data points and joining them. It is really only the actual data points that count, but sometimes by joining them you get a better impression of the trend in the data points. This method of representation is particularly useful when illustrating a trend over time.

Worked Example 1

As part of a science project Evan records the height of a plant every week. His results are in this table.

Week	0	1	2	3	4	5	6
Height (cm)	0	1	3	4	6	8	9

Draw a line graph.

Solution

First draw a suitable set of axes.

Then plot a point for each measurement as shown below.
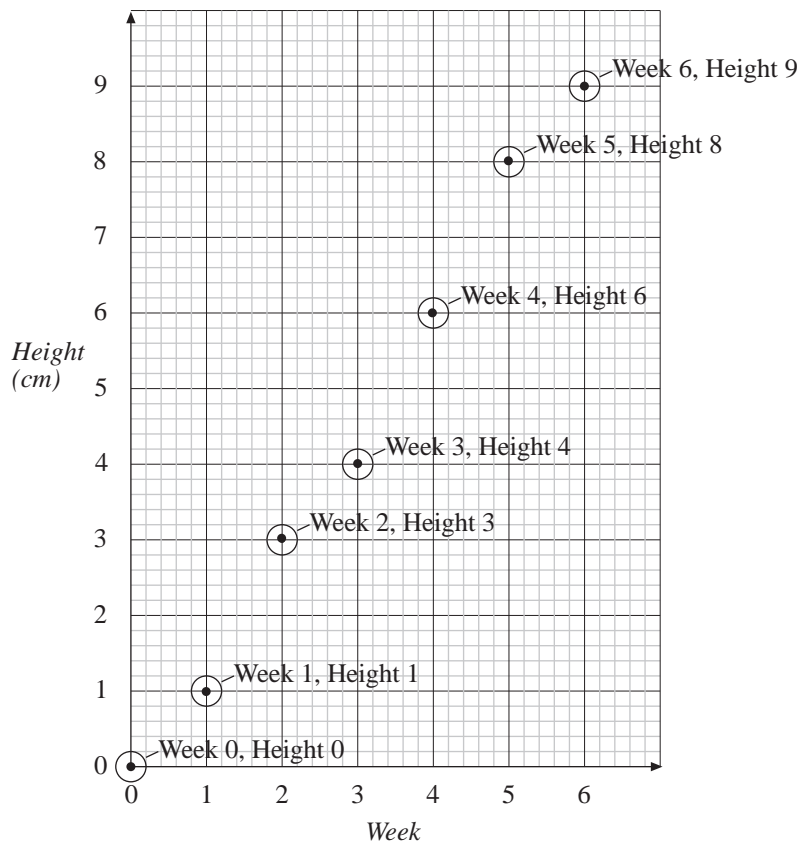
The points can then be joined with straight lines as shown in the next graph.

2.5
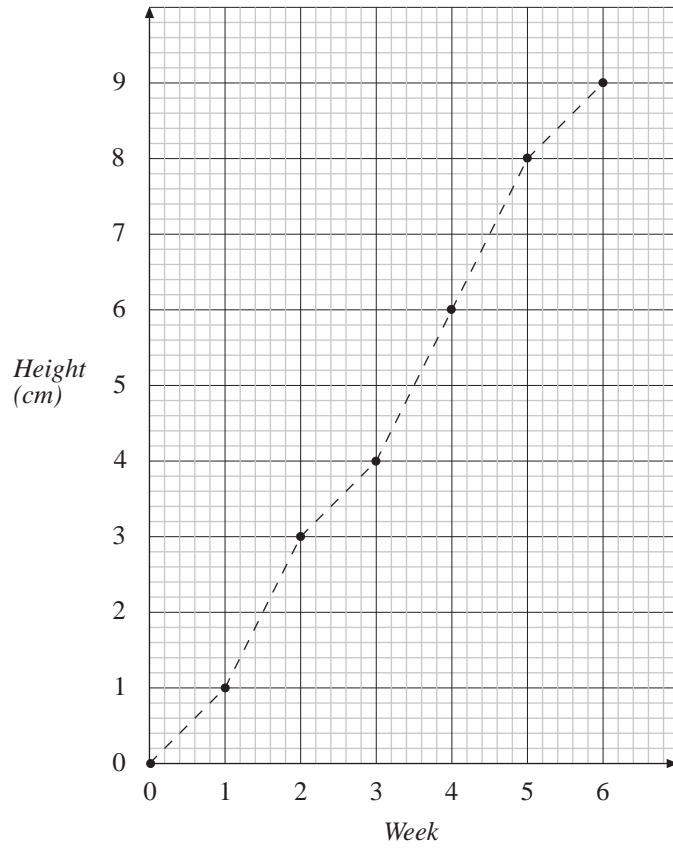
Worked Example 2

Stuart recorded the temperature in his greenhouse at 6 pm each day for a week. His records are shown on this line graph.
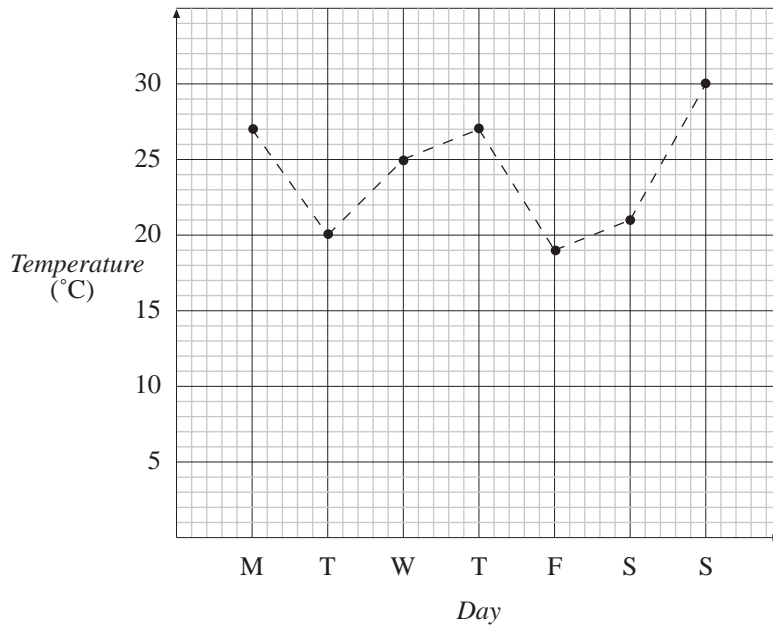
(a)   What was the temperature on Wednesday?

(b)   What was the lowest temperature recorded?

(c)   What was the highest temperature recorded?

2.5

Solution

(a)   For Wednesday the temperature can be read as $25°C$ (see Diagram 1).

(b)   The lowest temperature occurred on Friday and was $19°C$ (see Diagram 2).

(c)   The highest temperature occurred on Sunday and was $30°C$ (see Diagram 2).

Diagram 1
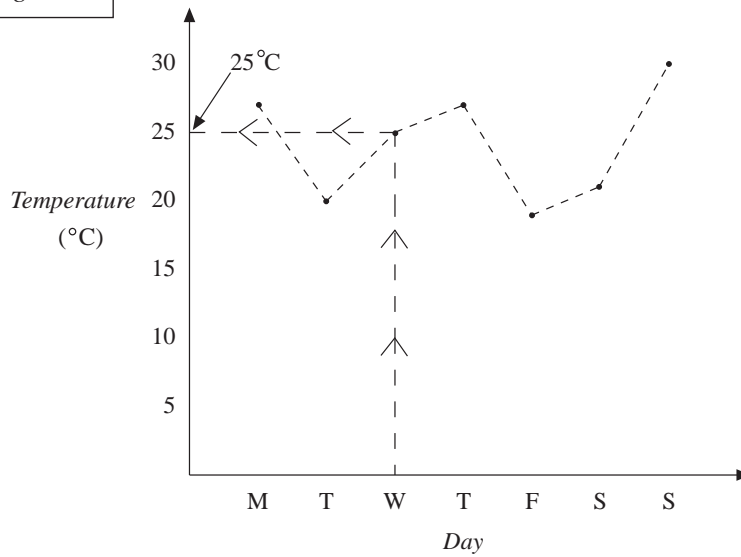
Diagram 2
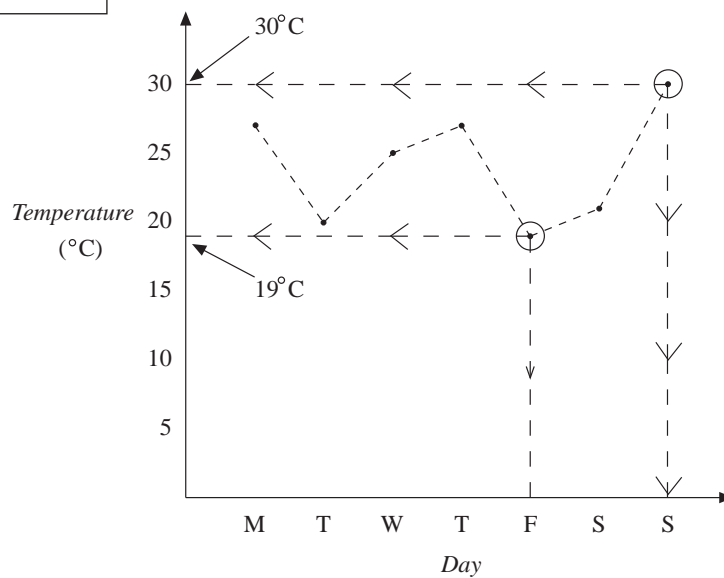
2.5

Exercises

1.   A mug was filled with hot water and the temperature was recorded every 5 minutes. The graph below shows the results.
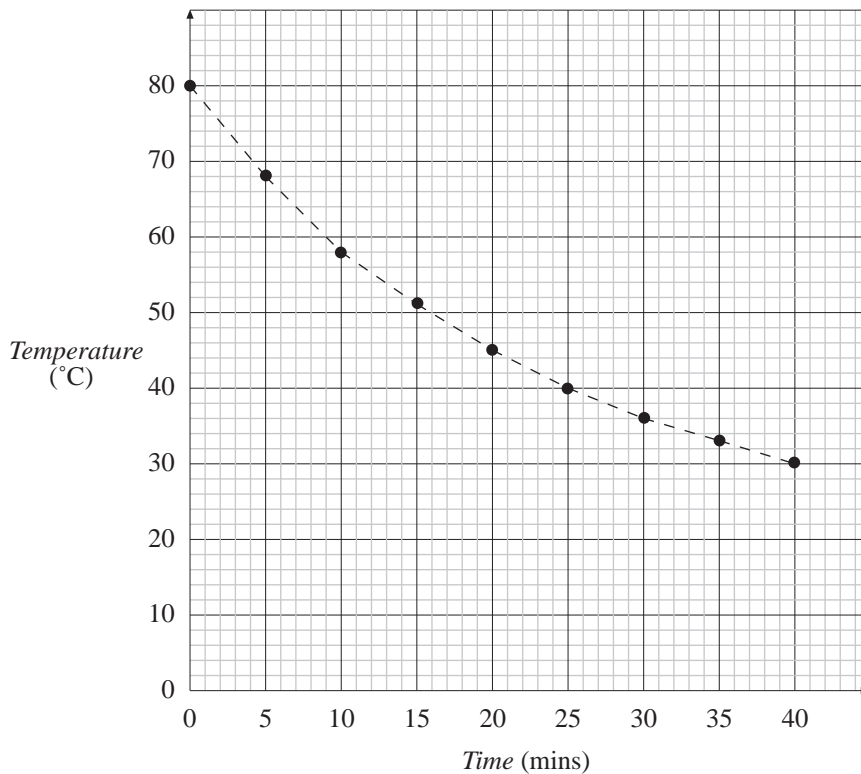
(a)   What was the temperature after 25 minutes?

(b)   What was the temperature at the start of the experiment?

(c)   When was the temperature $45°C$?

(d)   How long did it take for the temperature to drop from $68°C$ to $36°C$?

(e)   What would the temperature be after 1 hour?

(f)   What is happening to the rate of fall in the temperature of the water?

(g)   Will the temperature continue to fall? If not, why not?

2.   The following graph shows how the height of a sunflower plant changed since it was planted in a garden.

(a)   What was the height of the plant when it was planted in the garden?

(b)   How much did the plant grow in the first week?

(c)   What is the greatest height that the graph shows?

(d)   What do you think the eventual height of the sunflower will be?

(e)   How long did it take for the height to increase from 54 cm to 78 cm?

2.5
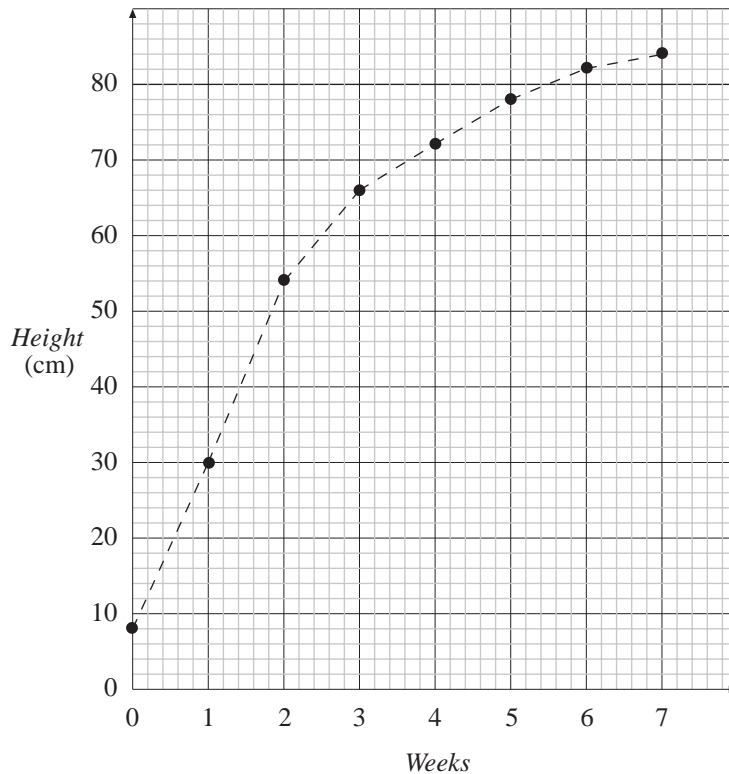
3.   Paul recorded the temperature outside his house at 8.00 am every day. His results
     are in the table.

Day	M	T	W	T	F	S	S
Temperature (°C)	8	5	4	6	7	5	3

Draw a line graph for this data.

4.   Karen counted the number of cars that drove past her while she was waiting at the
     bus stop each morning on her way to work.

Day	M	T	W	T	F	S
Number of cars	18	12	22	36	4	10

(a)   Draw a line graph for this data.

(b)   Give reasons for the wide variation in the number of cars Karen counted
      each morning while waiting for her bus.

5.   Anna recorded the time it took her to walk to school every day for a week.

Day	M	T	W	T	F
Time taken (mins)	8	9	15	12	7

(a)   Draw a line graph for this data.

(b)   Give a possible reason for Anna taking longer to walk to school on
      Wednesday and Thursday.

2.5

6. Stuart is training to run a marathon. Each week he recorded the time it took him to run 5 miles.

Week	0	1	2	3	4	5	6
Time (mins)	56	51	47	44	42	40	39

(a) Draw a line graph for this data.

(b) What is happening to his training times?

(c) What do you think his time for the five miles will be when he has completed his training?

2.6 Obtaining Data from Graphs and Charts

Data is presented in all sorts of ways, sometimes in misleading ways as will be seen later. We first look at examples of interrogating databases, whether in tabular, graphical or numerical form.

Worked Example 1

A life expectancy curve shows how many more years a person of a certain age is expected to live. The curve below shows life expectancy in Ruritania for men and women. For example, a 30-year-old man living there can expect to live another 34 years.
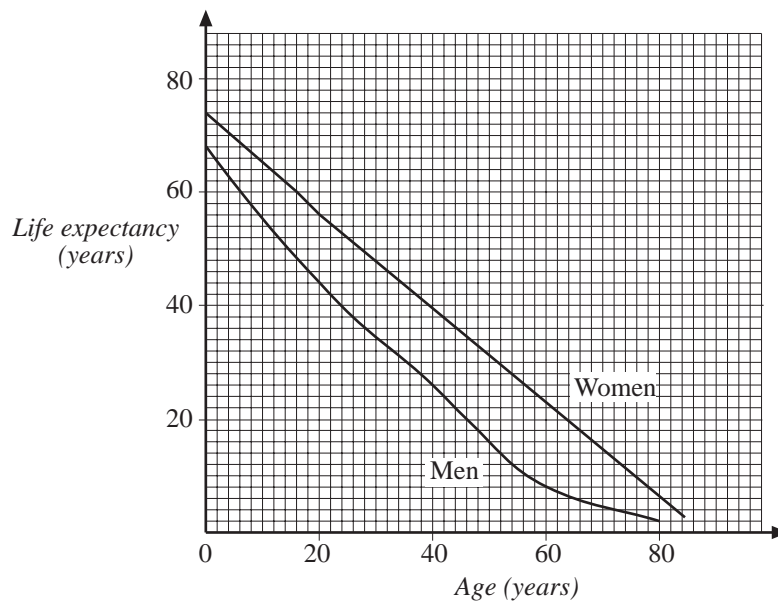
(a)   How many more years can a woman, aged 40, living there expect to live?

(b)   Winston is 60 years old and living in Ruritania. To what age can he expect to live?

(c)   Rula, a woman in Ruritania, has a life expectancy of 15 years. How old is Rula now?

(d)   At 40 years of age what is the difference in life expectancy for men and women?

(NEAB)

Solution

(a)   39 years

(b)   68 years

(c)   69 years

(d)   13 years

2.6

Worked Example 2

How long school children spend on their homework each evening
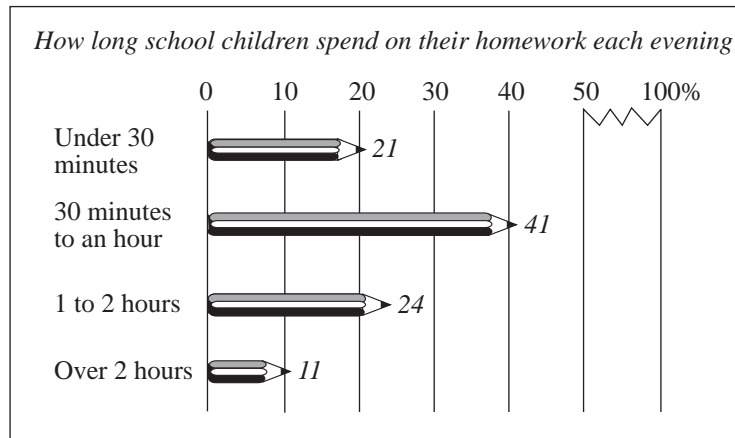
Source: *Which Magazine* October 1993

The above information was part of a survey, by questionnaire, given to 400 pupils.

(a)   Calculate the number of pupils who said that they worked more than two hours every evening

(b)   What percentage of the pupils did not answer this part of the questionnaire?

(c)   Give a suitable reason why these children may not have recorded an answer to this part of the questionnaire.

(SEG)

Solution

(a)   11% of 400   i.e. $400 \times \dfrac{11}{100} = 44$

(b) $100 - (21 + 41 + 24 + 11) = 100 - 97 = 3$,   i.e. 3%

(c)   Pupils who never do any homework may have left that part of the questionnaire blank because they thought that the first category 'under 30 minutes' did not apply to them.

Worked Example 3

Read the newspaper extract carefully. Use the information given in the extract to answer the following questions.

THE FACTS OF YOUNG LIFE

There are 11.8 million children in Britain, 17 per cent fewer than in 1971. The number is expected to increase by nearly 5 per cent by the end of the century as the babies born in the birth boom of the 1960s become parents themselves.

Children make up 20 per cent of the population compared with 30 per cent in 1911.

The lowest number of babies born since the Second World War was 600 000 in 1977. In 1992 there were 781 000 births and this figure is expected to increase each year to 1996 when the baby boom generation passes its child-bearing peak.

Babies are slightly more likely to be a boy than a girl: there are 105 males born for every 100 females.

2.6

(a) Is it expected that the number of children in Britain will have increased or decreased by the year 2000?

(b) If 500 people in Britain were chosen at random, how many of them would you expect to be children?

(c) What is the difference between the number of babies born in 1977 and the number born in 1992?

(d) Using the newspaper information, what is the probability that a baby is a boy?

Solution

(a) The number of children in Britain is expected to have increased by nearly 5% by the year 2000.

(b) 20% of $500 = 500 \times \dfrac{20}{100} = 100$ children

(c) $781\,000 - 600\,000 = 181\,000$ babies

(d) $\text{probability} = \dfrac{\text{no. of male babies}}{\text{total no. of babies}} = \dfrac{105}{205} \approx 0.512$

Worked Example 4

A tinned fruit manufacturer thought that people had difficulty identifying certain fruits just by taste.

To find out if this was true she conducted a taste experiment.

90 people were chosen and blindfolded.

30 were given plums, 30 were given prunes and 30 were given damsons.

The results are shown below.

		What people thought they were tasting		
		PLUMS	PRUNES	DAMSONS
What people were really tasting	PLUMS	18	1	11
	PRUNES	0	29	1
	DAMSONS	12	8	10

This table shows, for example, that one person tasted plums and thought they were prunes.

(a) How many people correctly identified damsons?

(b) How many tasted damsons and thought they were tasting plums?

(c) Which two fruits were most often confused?

(d) How can you tell that most people were not guessing?

Solution

(a) 10   (b) 12   (c) Plums and damsons

(d) If they were guessing, you would expect the number of people thinking they were tasting plums, prunes and damsons to be approximately the same.

2.6

Exercises

1. The pictogram shows the number of cars of different colours sold by a large
    garage during a period of 20 weeks.

    represents 10 cars

    Red

    White

    Blue

    Grey
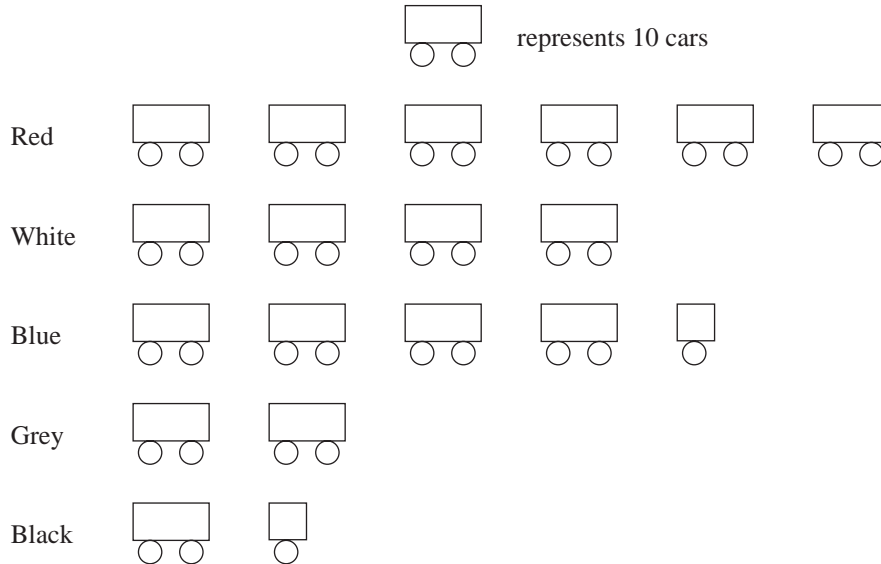
    Black

    (a)   How many white cars were sold?

    (b)   How many more blue cars than black cars were sold?

    (c)   Find the total number of cars sold.

    (d)   Calculate the mean number of cars sold per week.

    (e)   The profit made per car is £500. Calculate the total profit on the black cars.

    (NEAB)

2. The diagram shows the amount of energy needed by each kilogram of body weight
    each day between birth and the age of 18 years.

2.6

(a)   A boy is 15 years old. How much energy in kilojoules does he need each day for each kilogram of his body weight?

(b)   A girl is 16 years old. She weighs 50 kg. How much energy in kilojoules does she need each day?

(c)   What happens to the energy needs of the body as people grow older?

(SEG)

3.   The diagram below shows the soldier-teacher ratios for some Third World countries together with the United States and the United Kingdom.
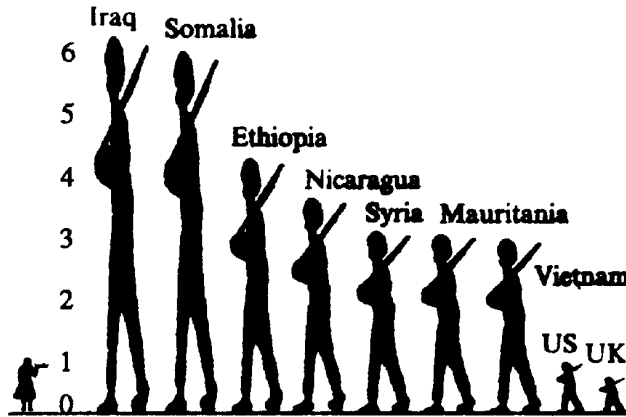
(a)   In Syria there are 600 000 teachers. How many soldiers are there?

(b)   In which countries are there more teachers than soldiers?

(c)   In Nicaragua there are 700 000 soldiers. How many teachers are there?

(d)   There are the same number of soldiers in Somalia as in Vietnam. What can you say about the number of teachers in Somalia and Vietnam?

(NEAB)

4.   Two students were each asked to collect statistical data. The information they collected is shown.

How students travel to school

Bus   ♀ ♀ ♀ ♀ ♀ ♀

Cycle ♀ ♀ ♀

Walk   ♀ ♀ ♀ ♀ ♀ ♀

Car   ♀ ♀ ♀ ♀ ♀ —

Train ⊥
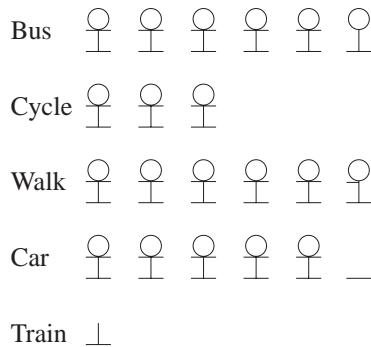
How many brothers and sisters students have

Brothers					
4	1	3			
3				1	
2	9	6			
1	17	14		1	
0	25	15	7		1
	0	1	2	3	4

Sisters

(a)   Fifteen students cycle to school. How many students walk?

(b)   How many students have got two brothers and one sister?

(c)   How many children are in the largest family?

(SEG)

2.6

5.

THE WORLD'S HEAVIEST SMOKERS			
Annual cigarette consumption per country (millions). All figures are for 1991.		Daily cigarette consumption per man, woman and child. All figures are for 1991.	
China	1 617 000	Greece	7.8
USA	516 500	Japan	7.3
CIS and the Baltic States	456 000	Poland	7.3
Japan	328 300	Hungary	7.0
Brazil	156 400	Switzerland	6.5
Indonesia	146 511	Bulgaria	6.1
Germany	146 500	South Korea	6.0
Poland	102 100	Spain	5.9
France	97 100	Australia	5.6
United Kingdom	96 838	USA	5.6
		United Kingdom = 4.6	

(a)   How many more cigarettes were smoked in the USA than in Germany during 1991?

(b)   A typical smoker in the USA was given 84 cigarettes.

How long would you expect these cigarettes to last?

(c)   State the reason why China can be top of the consumption table and yet the consumption per person is not recorded on the table.

(SEG)

6.   The bar chart shows the number of children buying different types of food from the canteen on a particular day.
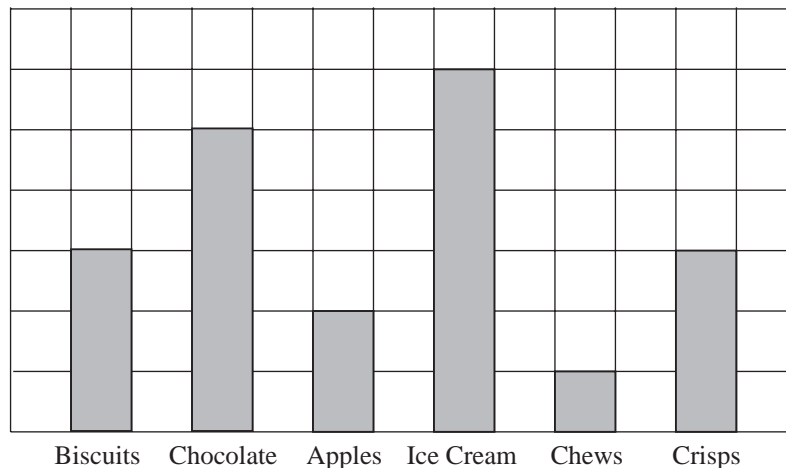
Chocolate was bought by 30 children.

(a)   Work out the number of children who bought chews.

(b)   Find the number of children who bought ice cream.

Each child buys only one type of food.

(c)   Calculate the total number of children buying food on that day.

2.6

Another form of presentation was proposed.

(d)   Keeping the foods in the same order as on the bar chart complete a percentage bar chart in a copy of the one below.

Biscuits	
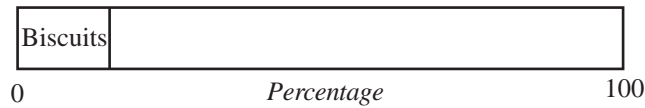
0   *Percentage*   100

(SEG)

7.   The population pyramids for India and the United Kingdom show the percentage of males and females within each age group.
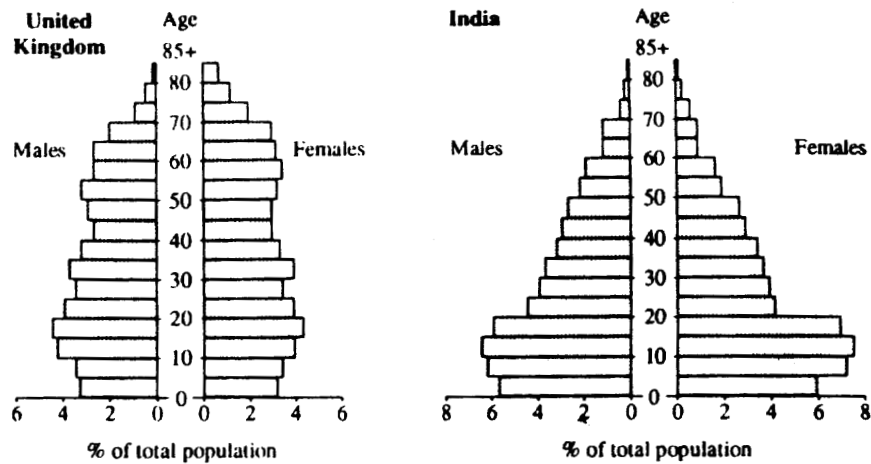
(a)   Which age group of males made up 2% of the population in the United Kingdom?

(b)   (i)   Which age group in India accounted for the highest percentage of the population?

         (ii)   Estimate what percentage of the population of India is in this age group.

(c)   Estimate the percentage of the population of the United Kingdom who were female and less than 10 years old at the time the information was collected.

(d)   Give *two* comments on the population structure in these countries for people over 70 years of age.

| 2.7 | # Misleading Diagrams |

The media (both the press and TV) often misrepresent data in order to unfairly emphasise a particular point. You will see some instances of this in the following examples.

Worked Example 1

The following diagram shows how the average price of a house has increased in less than 2 years.

£61 520   £72 800
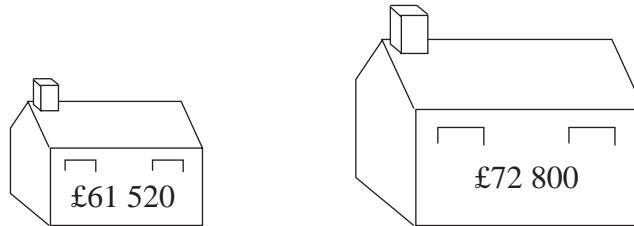
Explain why the diagram is misleading.

Solution

The ratio of the house prices

$$= £100\,000 : £120\,000$$

$$= 1 : 1.2$$

In the diagram, the larger house is 1.5 times longer than the smaller one, which means that the volume of the larger house is more than 3 times the volume of the smaller house.

This means that the sizes of the houses are not in the same ratio as their prices.

Worked Example 2

BIGGER BOTTLES

– smaller prices

70cl
£6.65

1 litre
£9.15

Old size   New size

(a) Explain why the slogan "BIGGER BOTTLES – smaller prices" on the advertisement could be misleading.

(b) Explain why this advertisement is correct. *(SEG)*

2.7

Solution

(a)   It sounds as if the bigger bottle is actually cheaper than the smaller bottle.

(b)   The cost per litre of the smaller bottle is

$$£6.65 \times \frac{100}{70} = £9.50$$

so, in fact, the bigger bottle has a smaller price for the equivalent volume.

Worked Example 3

The unusual diagram below was produced by a nature conservation group.
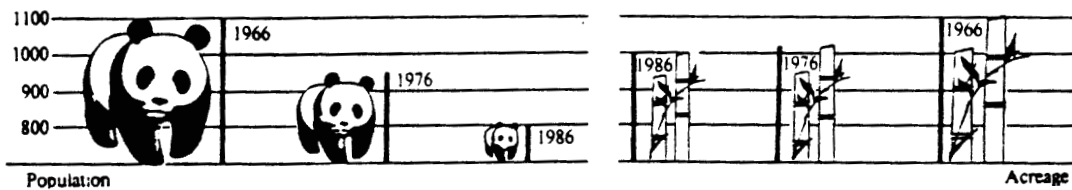
(a)   The panda population was smaller in 1986 than in 1966.
        Approximately how much smaller?

(b)   Give *two* ways in which the panda diagram is misleading.

(c)   Describe briefly the change in bamboo yield from 1966 to 1986.

(d)   What has been omitted from the bamboo diagram?

(e)   Name *one* unusual feature of the bamboo yield diagram that might lead to
        misunderstanding.

(NEAB)

Solution

(a)   About 300 less.

(b)   The vertical axis starts at 700, so the decrease looks larger than it really is; also, the
        sizes of the pandas are not in proportion to 1100 : 950 : 800.

(c)   Significant decreases from 1966 to 1976, but little change from 1976 to 1986.

(d)   The scale on the vertical axis.

(e)   The years are in the opposite order to the panda population diagram, so are not in
        chronological order.

2.7

Exercises

1.

EXCELLENT PAYOUT

The Fairplan Company	Average	Worst Company
£71 317	£67 006	£51 374
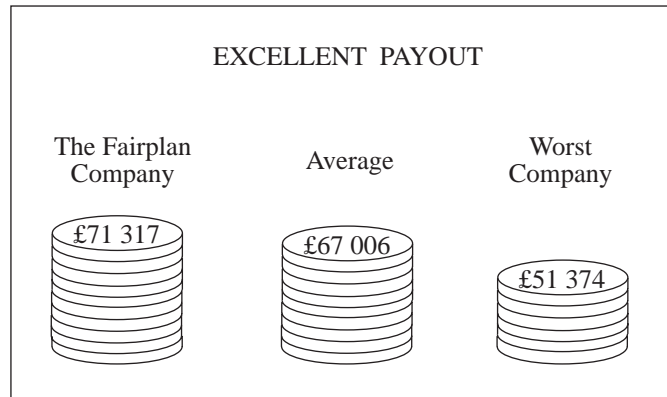
The pictogram shows the amount of pension given to people when they retire.

(a)   Calculate the amount of money that each disc represents for the Worst Company.

Give your answer to the nearest £.

(b)   Why could it be misleading to compare The Fairplan Company with the Worst Company using this pictogram?

(SEG)

2.   The diagram below shows the percentage of British people who own their own house.

Growth of owner occupation as a % of all British householders
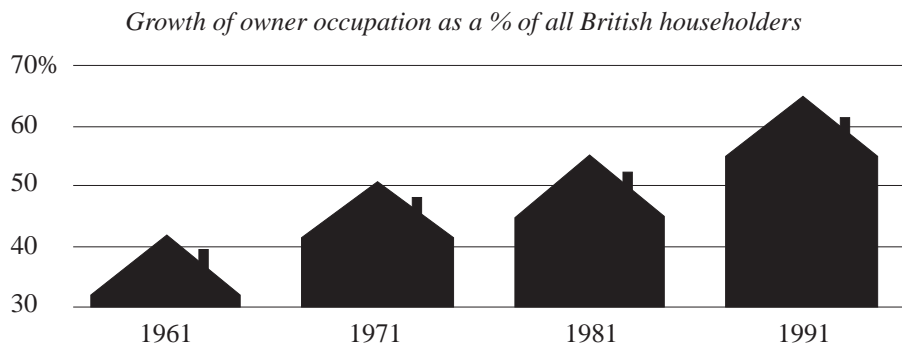
State two ways in which the diagram is misleading.

(NEAB)

2.7

3. The diagram shows the number of visits (in millions) made to the United Kingdom by overseas residents for the years from 1979 to 1989.

Visits to the UK by overseas residents
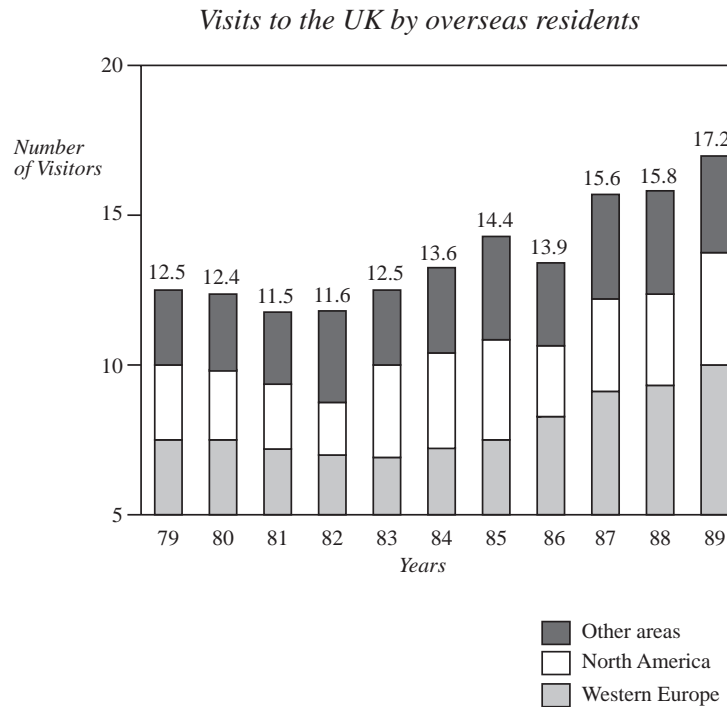
Source: *International Passenger Survey*

(a) Give two reasons why the diagram is misleading.

(b) In which year did the total number of visitors exceed 14 million for the first time?

(c) Estimate, to the nearest million, the number of visitors from Western Europe in 1983.

(d) In which year did the total number of visitors from Western Europe and North America exceed 13.6 million?

(NEAB)

4. The following diagram relates to the SNCF (French railways) and the railways in West Germany and the United Kingdom.

(a) In which year did the SNCF make its greatest operating loss?

(b) How much was the operating loss in 1982?

(c) Describe the change in productivity of the three railways between 1960 and 1985.

(d) Why is the graph of 'productivity' misleading?

2.7

F Fr bn **Operating profit/losses**

1979 80 81   84 85 86 est

Productivity

Source: Union Internationale des Chemins de Fer
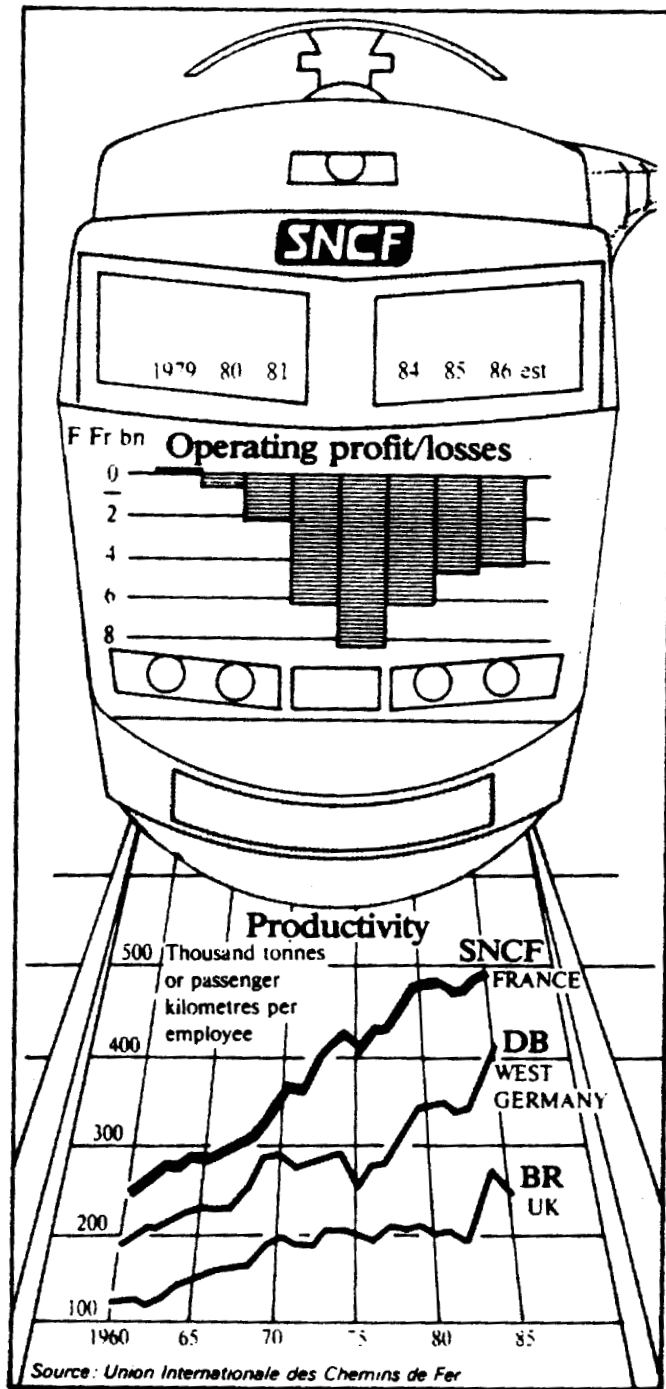
(NEAB)

2.8 Frequency Graphs

For *continuous data*, when any value over a range of values is possible, a *frequency graph* like the one below should be used, rather than a bar chart which is used for discrete data.
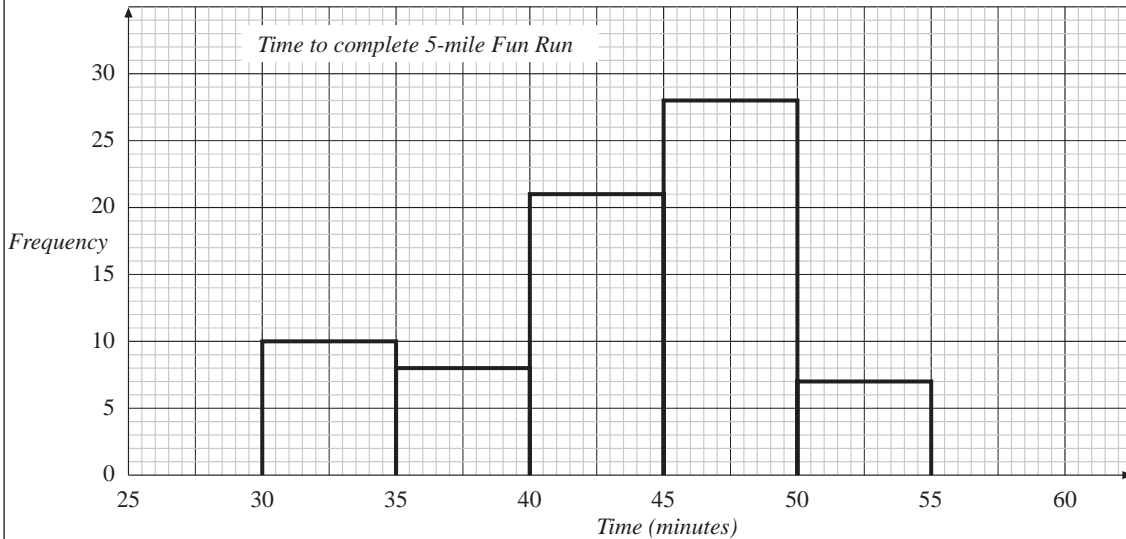
A graph like this is often called a *histogram,* and is characterised by having a continuous scale along the horizontal axis. Note that in this case the widths of the bars are all the same, but this is not always the case, as you will see in the section on histograms. Care though must be taken about the end points. For example, the first *class interval* (in minutes) would normally be $30 \leq \text{time} < 35$, so that a time of 35 minutes would be in the second class interval.

A *frequency polygon* could also be used to show the same data, as on the following graph. Note how it is related to the histogram in that the points are plotted in the centre of the class intervals.
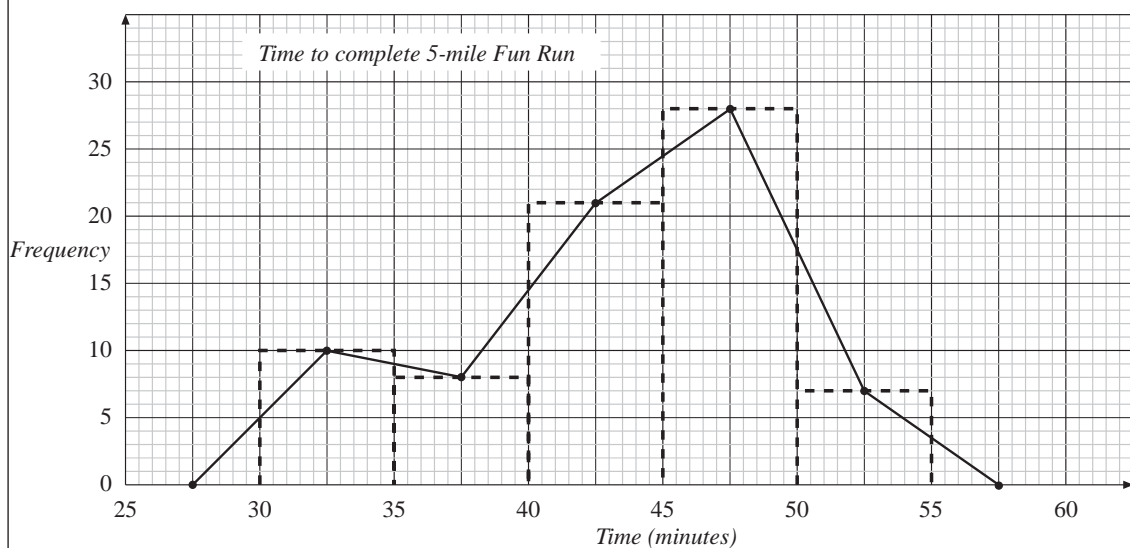
2.8

Worked Example 1

Use the data shown on the graphs above to answer these questions.

(a)   How many people completed the Fun Run in between 40 and 45 minutes?

(b)   How many people completed the Fun Run in less than 40 minutes?

(c)   How many people completed the Fun Run in less than 1 hour?

Solution

(a)   The 40-45 minute interval contains 21 people.

(b)   The 30-35 and 35-40 minute intervals must be considered.
       There are 10 people in the 30-35 minute interval.
       There are 8 people in the 35-40 minute interval.
       So there are $10 + 8 = 18$ people who complete the run in less than 40 minutes.

(c)   The number in each interval is needed.
       So the number of people is:
$$10 + 8 + 21 + 28 + 7 = 74$$

Worked Example 2

A group of students measured the reaction times of 50 other students. The times are given below correct to nearest hundredth of a second.

0.44	0.32	0.31	0.47	0.27	0.31	0.40	0.28	0.16	0.26
0.33	0.46	0.41	0.33	0.31	0.28	0.38	0.29	0.17	0.26
0.29	0.40	0.29	0.24	0.41	0.22	0.25	0.47	0.31	0.36
0.49	0.21	0.42	0.43	0.28	0.36	0.24	0.37	0.34	0.27
0.49	0.16	0.29	0.30	0.41	0.27	0.29	0.28	0.40	0.42

Draw a histogram for this data.

Solution

First the data must be collected into groups, using a tally chart. The class intervals should be chosen to give sufficient groups; in most circumstances this will be between 5 and 10 groups.

Reaction Time(s)	Tally	Frequency
$0.15 \le t < 0.20$	|||	3
$0.20 \le t < 0.25$	||||	4
$0.25 \le t < 0.30$	卌 卌 卌	15
$0.30 \le t < 0.35$	卌 ||||	9
$0.35 \le t < 0.40$	||||	4
$0.40 \le t < 0.45$	卌 卌	10
$0.45 \le t < 0.50$	卌	5

Now that the data has been collected in this way, the following histogram can be drawn.
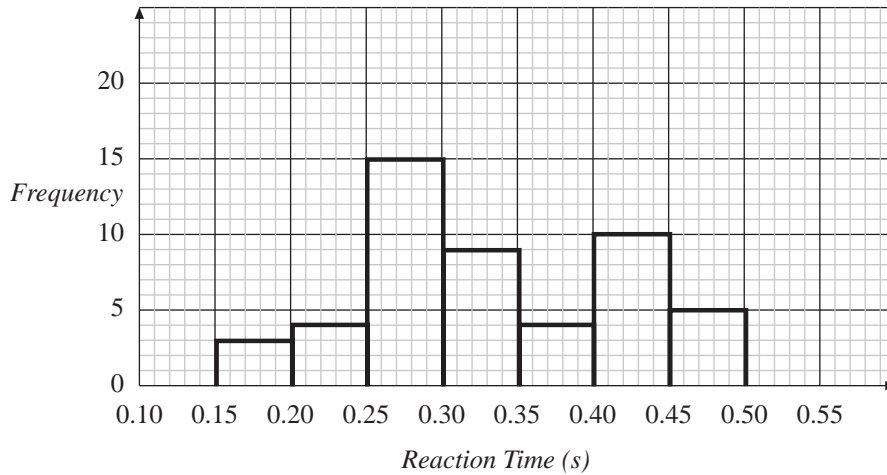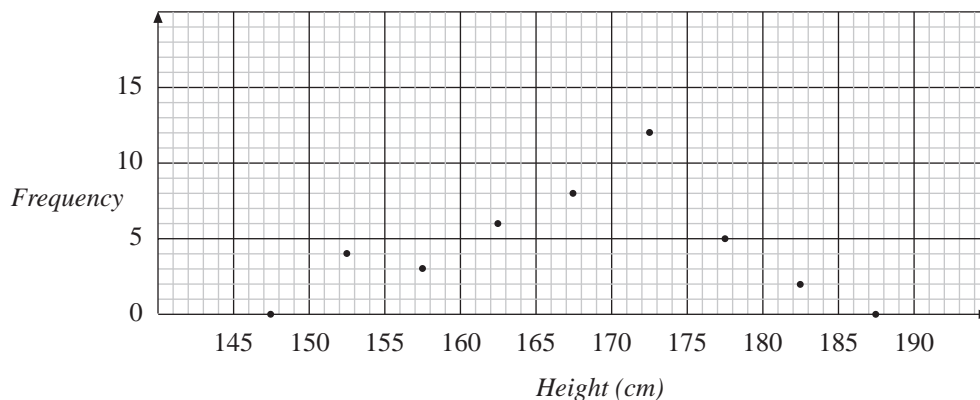
Worked Example 3

Draw a frequency polygon for the data on the height of children given in the table below.

| Height (cm)	Frequency
$150 \leq h < 155$	4
$155 \leq h < 160$	3
$160 \leq h < 165$	6
$165 \leq h < 170$	8
$170 \leq h < 175$	12
$175 \leq h < 180$	5
$180 \leq h < 185$	2

Solution

Points should be placed above the centre of each interval. The height is given by the frequency. The following graph shows these points.

Note hat points have been placed on the horizontal axis in the middle of the class intervals immediately before and after the given group of intervals. These points represent a frequency of 0 for each of the class intervals $145 \text{ cm} \leq h < 150 \text{ cm}$ and $185 \text{ cm} \leq h < 190 \text{ cm}$. Points have now been plotted in the middle of each class interval.

2.8

The points are then joined to give the following frequency polygon.

2.8
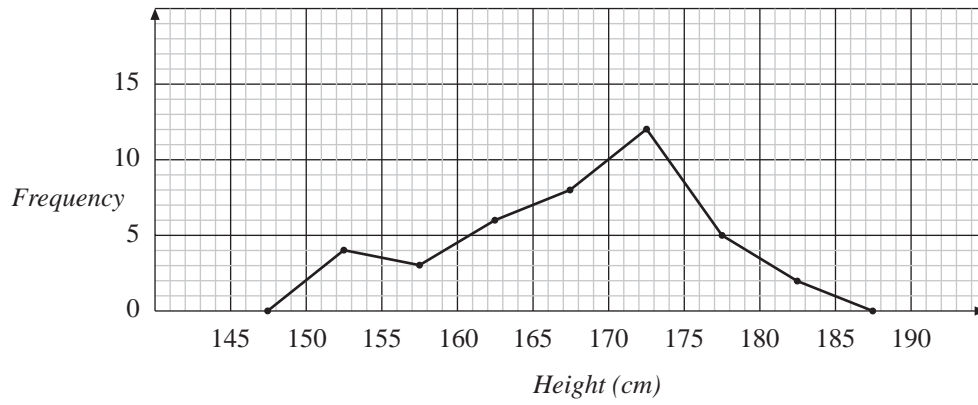
Exercises

1. The histogram below shows how the weights of children in one year group were distributed.
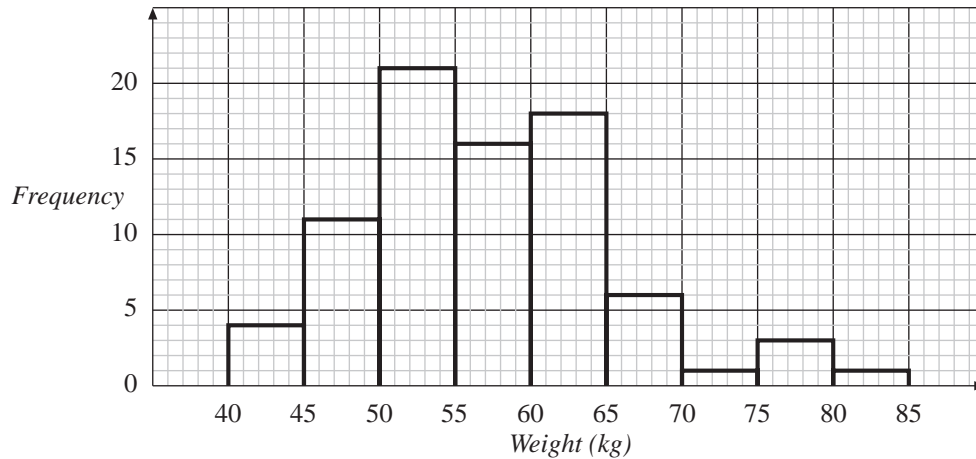
    (a) How many children had a weight greater than 70 kg?

    (b) How many children had a weight between 50 and 65 kg?

    (c) How many children had a weight less than 50 kg?

    (d) How many children were there in the year group?

2. The frequency polygon shows the weekly wages of a large firm.
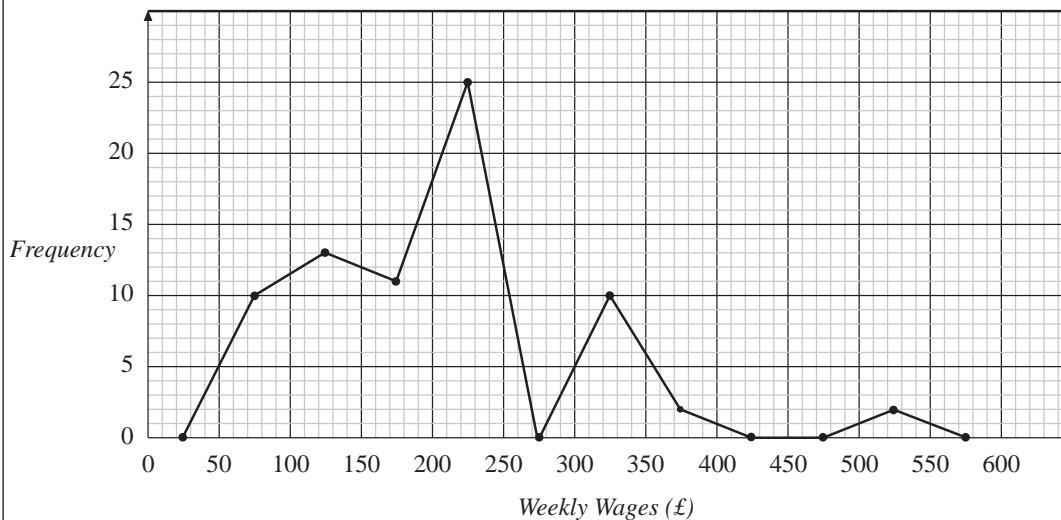
    (a) How many people earn between £300 and £350 per week?

    (b) How many people earn between £100 and £300 per week?

    (c) How many people are employed by the firm?

    (d) What are the largest and smallest possible weekly wages that the graph shows could be paid?

    (e) This frequency polygon shows three peaks (of different heights). Can you suggest reasons for this distribution of wages?

2.8

3. An orchard contains 100 apple trees. The weight of apples produced by each tree in one year was recorded.

    The results are given in the table .

    Draw a histogram for the data.

Mass of apples (kg)	Frequency
$50 < m \leq 60$	5
$60 < m \leq 70$	7
$70 < m \leq 80$	13
$80 < m \leq 90$	10
$90 < m \leq 100$	20
$100 < m \leq 110$	22
$110 < m \leq 120$	18
$120 < m \leq 130$	5

4. A psychologist uses a test in which people have to solve a puzzle. He records the time it took people to solve the puzzle.

Time taken (mins)	$0 \leq t < 1$	$1 \leq t < 2$	$2 \leq t < 3$	$3 \leq t < 4$	$4 \leq t < 5$
Frequency	5	32	18	7	12

    (a) Draw a histogram for the data.

    (b) Describe what the histogram shows.

5. The finishing times for a cross country race were recorded to the nearest minute.

    Draw a suitable histogram for the data.

23	38	43	47
27	39	43	48
31	39	43	48
32	40	43	48
32	40	44	50
32	40	44	50
33	41	46	51
34	41	46	51
35	42	46	52
37	42	47	53

6. At the end of a football season a newspaper calculated the mean number of goals scored per match for 100 top footballers.

2.7	1.4	2.9	1.3	2.1	2.1	2.9	2.1	1.7	3.1
1.2	2.0	0.6	2.2	2.2	2.2	2.7	1.6	1.3	1.9
1.3	1.4	1.9	2.1	1.8	0.9	2.1	2.3	1.9	1.3
1.3	0.3	1.2	2.0	1.4	1.7	2.4	1.9	1.7	1.5
2.7	2.2	0.7	1.5	1.6	1.4	2.7	2.0	1.9	2.7
2.1	1.6	0.7	1.5	2.0	0.9	1.6	1.6	1.5	2.6
1.1	2.2	1.8	1.5	1.8	2.6	0.2	1.2	1.4	1.9
1.7	1.4	2.1	2.2	1.5	2.1	2.4	0.5	0.9	1.4
1.8	0.6	1.9	2.6	1.6	2.1	2.0	1.8	1.3	2.0
1.2	2.2	1.4	2.1	1.2	0.4	2.6	1.9	0.9	2.1

    Use the data given to draw a suitable histogram and then draw a frequency polygon on top of the histogram.

2.8

7. Two students recorded the time it took drivers of cars to find a space and park in a car park. They also noted if the drivers were male or female.

Time Taken (mins)	$0 < t \leq 2$	$2 < t \leq 4$	$4 < t \leq 6$	$6 < t \leq 8$	$8 < t \leq 10$	$10 < t \leq 12$
Male Drivers	4	11	24	11	3	2
Female Drivers	0	16	21	5	5	8

(a) On the same set of axes draw frequency polygons for male and female drivers.

(b) What evidence does the graph provide to support the claim that females take longer to find a space and park than males?

8. The age distribution in a town is given in the table.
Draw a histogram to show the data.

Age	Frequency
$0 \leq a < 10$	1800
$10 \leq a < 20$	1500
$20 \leq a < 30$	1450
$30 \leq a < 40$	1600
$40 \leq a < 50$	1250
$50 \leq a < 60$	1150
$60 \leq a < 70$	800
$70 \leq a < 80$	500
$80 \leq a < 90$	150

9. A hire company owns three types of car which are classified as small, medium and large. The distance travelled by each car is always recorded.

Distance (miles)	$0 < t \leq 100$	$100 < t \leq 200$	$200 < t \leq 300$	$300 < t \leq 400$	$400 < t \leq 500$
Small Cars	80	50	30	2	1
Medium Cars	30	45	67	70	10
Large Cars	5	12	16	24	12

(a) On the same set of axes draw frequency polygons for each type of car.

(b) Comment on the graphs you have drawn.

10. A large number of children entered a mathematics competition. Their scores are listed below.

Score	$0 \leq s < 20$	$20 \leq s < 40$	$40 \leq s < 60$	$60 \leq s < 80$	$80 \leq s < 100$
Frequency	82	166	342	220	54

Draw a histogram for the data.

2.8

11. The graph shows the result of a survey of the times at which pupils arrive at school one day.
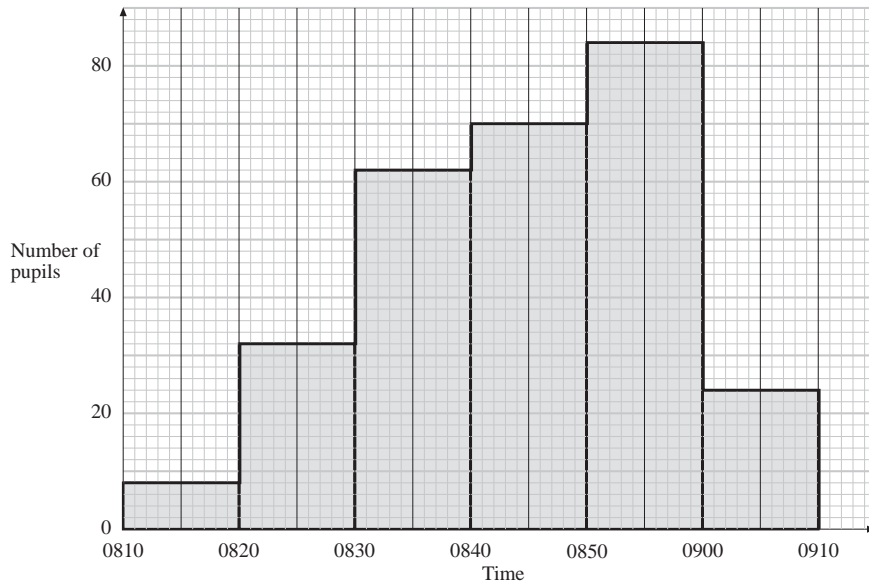
How many pupils arrived for school between 0830 and 0850? *(SEG)*

12. The table below gives information about the expected lifetimes, in hours, of 200 light bulbs.

Lifetime (t)	$0 < t \le 400$	$400 < t \le 800$	$800 < t \le 1200$	$1200 < t \le 1600$	$1600 < t \le 2000$
Frequency	32	56	90	16	6

(a) Mr Jones buys one of the light bulbs.

(i) What is the probability that it will not last more than 400 hours?

(ii) What is the probability that it will last at least 800 hours but not more than 1600 hours?

(b) Using axes similar to those below, draw a frequency polygon to illustrate the information in the table.
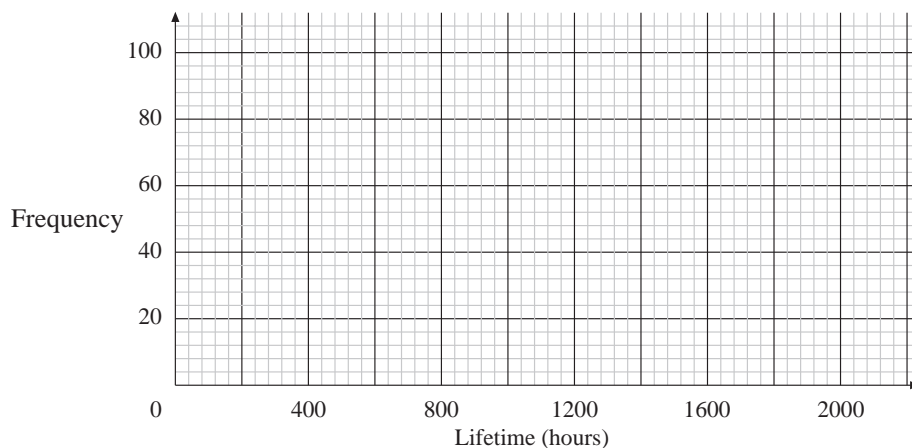
(MEG)

2.8

13. The height of some pupils is recorded.

Height h (cm)	Frequency
$120 \le h < 125$	2
$125 \le h < 130$	5
$130 \le h < 135$	8
$135 \le h < 140$	14
$140 \le h < 145$	11
$145 \le h < 150$	9
$150 \le h < 155$	3
$155 \le h < 160$	1

Ann records the data using class intervals of 10 cm.

(a) Copy and complete Ann's table.

Height h (cm)	Frequency
$120 \le h < 130$	
$130 \le h < 140$	
$140 \le h < 150$	
$150 \le h < 160$	

Ann draws a frequency diagram of her data.

Heights of pupils
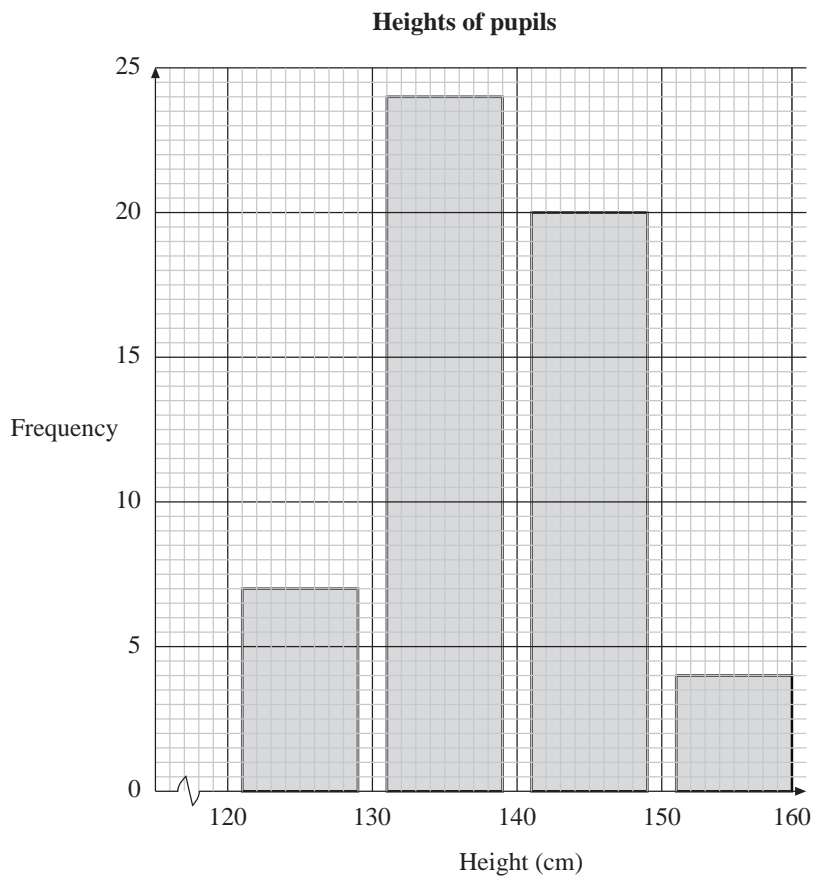
Ann has made two mistakes in drawing her diagram.

(b)   What are the two mistakes?

Another pupil is included.

The pupil has a height of 150 cm.

(c)   Into which of Ann's class intervals should the pupil be placed?

(SEG)

14.   The height of each of 60 plants of type *A* was measured and recorded.

Height of plant (cm)	8—	10—	12—	14—	16—	18—	20-22
Number of plants	0	2	3	18	19	18	0

(a)   Draw the frequency polygon of these results on a grid like the one below.
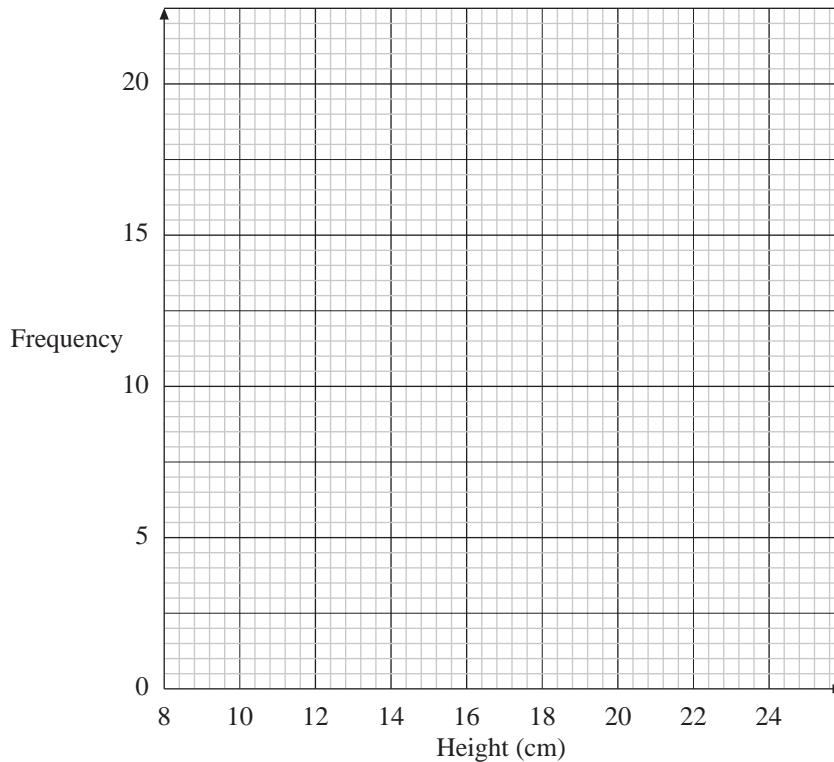
The following graph shows a frequency polygon of 60 plants of type *B*.

2.8
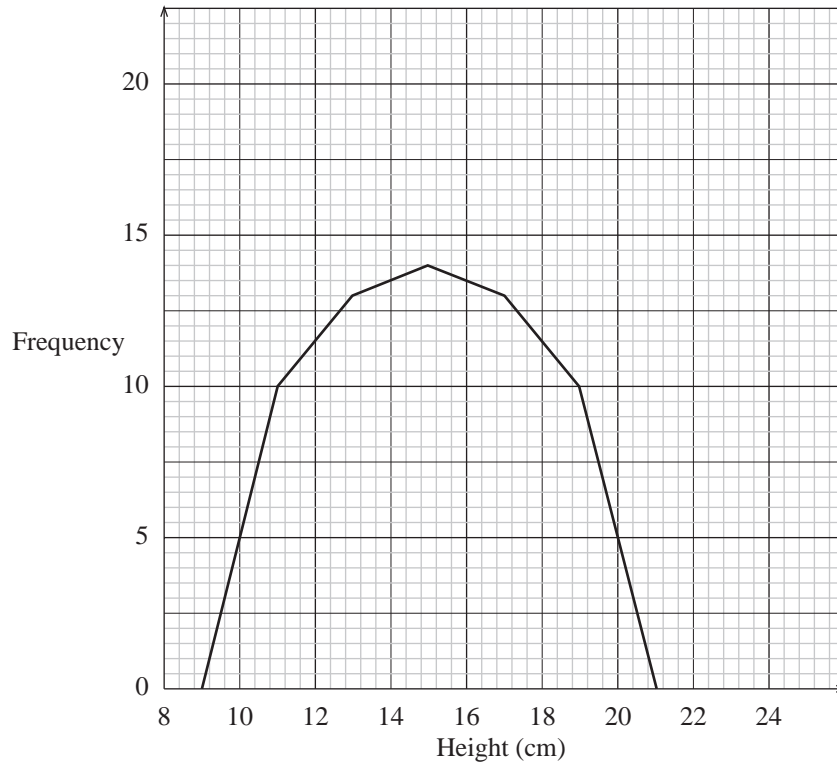
(b)   Write down **two** differences between the two types of plant shown by the frequency polygons.

(SEG)

2.9 Histograms

When drawing histograms it is possible that the intervals will not have the same width.

Consider the data given in the table opposite.

Weight (in grams)	Frequency
$0 \le w < 40$	5
$40 \le w < 50$	6
$50 \le w < 60$	8
$60 \le w < 70$	4
$70 \le w < 100$	2

The way the data have been presented makes it impossible to draw a histogram with equal class intervals.

In order to keep the histogram fair the *area* of the bars, rather than the height, must be proportional to the frequency. So on the vertical scale we plot *frequency density* instead of frequency, where

$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Class Width}}$$

Rewriting the table with an extra column for frequency density, gives

Weight (in grams)	Frequency	Frequency Density
$0 \le w < 40$	5	$\frac{5}{40} = 0.125$
$40 \le w < 50$	6	$\frac{6}{10} = 0.6$
$50 \le w < 60$	8	$\frac{8}{10} = 0.8$
$60 \le w < 70$	4	$\frac{4}{10} = 0.4$
$70 \le w < 100$	2	$\frac{2}{30} = 0.067$

and you can draw the histogram with frequency density on the vertical axis.
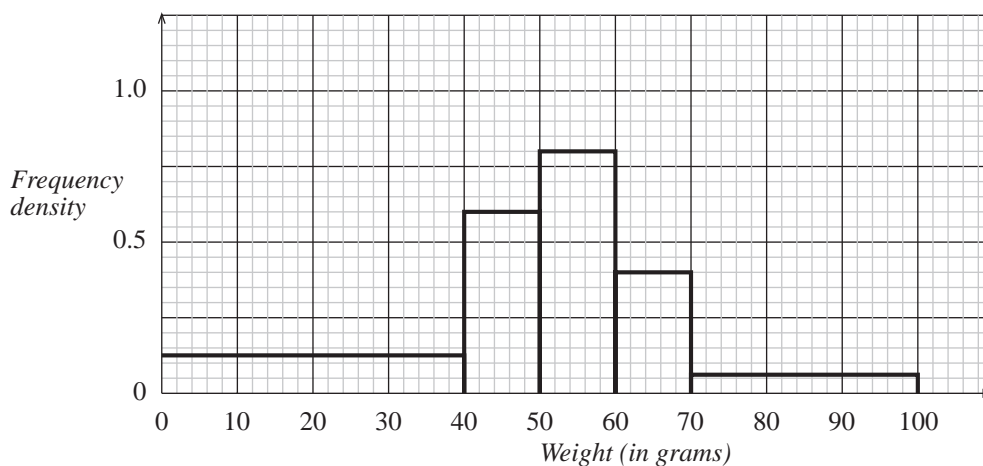
Note

You can see that it is the area that is proportional to the frequency – in fact, a frequency of 1 is represented by 10 little squares in this example.

2.9

Worked Example 1

Police officers recorded the speeds of vehicles passing a speed camera on an open road. The data is given in the table opposite.

Speed (mph)	Frequency
$0 \le v < 30$	3
$30 \le v < 40$	17
$40 \le v < 45$	21
$45 \le v < 50$	28
$50 \le v < 70$	32

(a)   Draw a histogram for this data.

(b)   If the speed limit on this section of road is 50 mph,

      (i)   How many motorists *may* be breaking the law?

      (ii)   How many motorists *must* be breaking the law?

      Explain your answers to (i) and (ii).

Solution

The following table shows how the frequency density can be calculated.

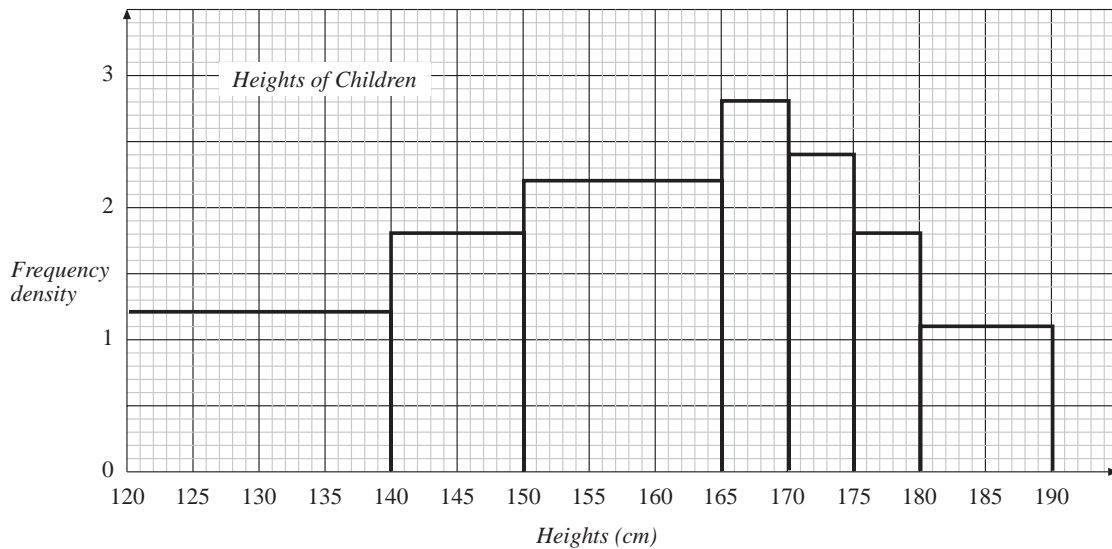| Speed (mph)	Class width	Frequency	Frequency Density
$0 \le v < 30$	30	3	$\dfrac{3}{30} = 0.1$
$30 \le v < 40$	10	17	$\dfrac{17}{10} = 1.7$
$40 \le v < 45$	5	21	$\dfrac{21}{5} = 4.2$
$45 \le v < 50$	5	28	$\dfrac{28}{5} = 5.6$
$50 \le v < 70$	20	32	$\dfrac{32}{20} = 1.6$

(a)   The histogram is shown below.
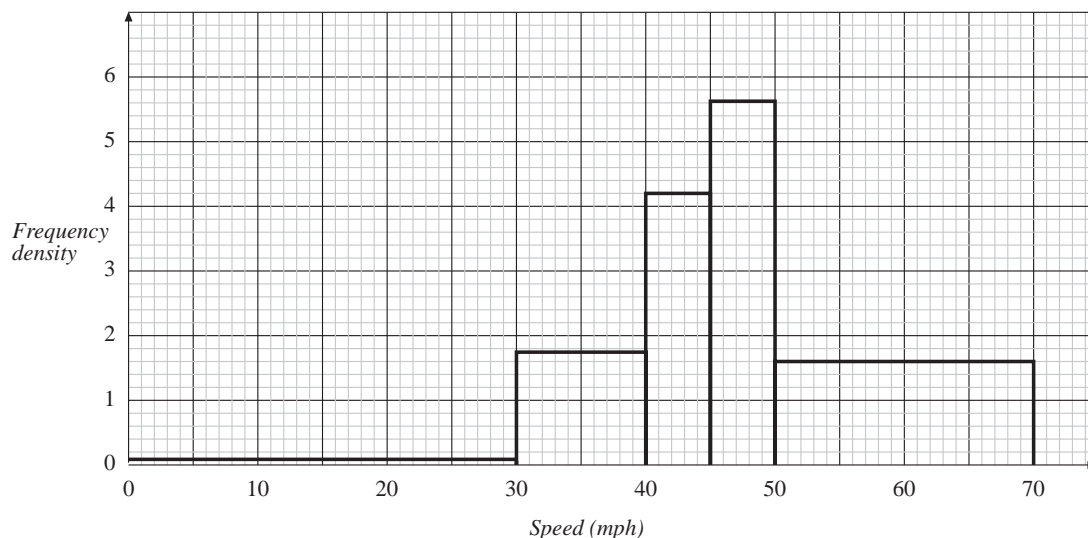
2.9

(b) (i) 32 motorists *may* be breaking the law. (All the motorists may have been driving at more then 50 mph, as this interval goes up to 70 mph.)

(ii) No motorists *must* have been breaking the law. (As the interval starts at 50 mph it is possible, although unlikely, that all of the motorists were driving at exactly 50 mph!)

Worked Example 2

The histogram below shows the results of a survey into the height of children in a school.

Heights of Children

Frequency density

Heights (cm)

(a) Find the number of children with heights between:

(i) 120 and 140 cm,   (ii) 170 and 175 cm.

(b) Find the total number of children measured.

Solution

(a) (i) For the 120 to 140 cm interval:

$$\text{Frequency Density} \;=\; 1.2$$
$$\text{Class Width} \;=\; 20$$
$$\text{Frequency} \;=\; 20 \times 1.2$$
$$=\; 24 \text{ children}$$

(ii) For the 170 to 175 cm interval:

$$\text{Frequency Density} \;=\; 2.4$$
$$\text{Class Width} \;=\; 5$$
$$\text{Frequency} \;=\; 5 \times 2.4$$
$$=\; 12 \text{ children}$$
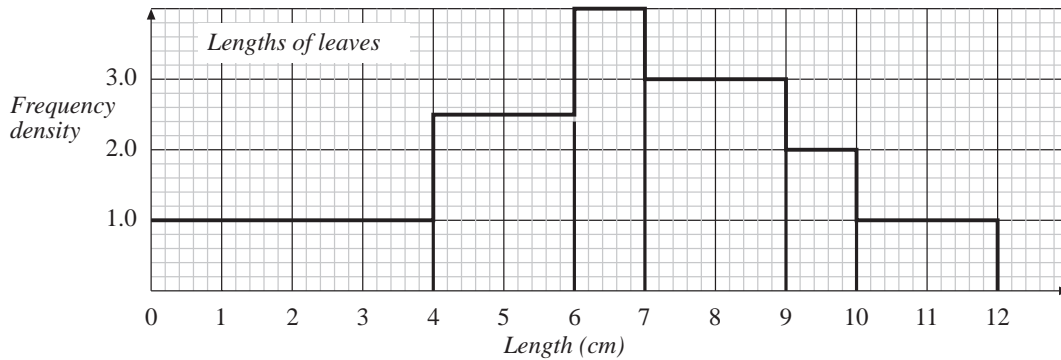
(b) To find the total, the numbers in each class interval must be found and added together.

$$\text{Total} = 20 \times 1.2 + 10 \times 1.8 + 15 \times 2.2 + 5 \times 2.8 + 5 \times 2.4 + 5 \times 1.8 + 10 \times 1.1$$

$$= 24 + 18 + 33 + 14 + 12 + 9 + 11$$

$$= 121$$

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Exercises

1. For a project in Biology, Sharma gathered data on the length of leaves from a tree and drew the histogram below.

    Lengths of leaves

    (a) How many leaves had a length less than 6 cm?

    (b) How many leaves had a length greater than 9 cm?

    (c) How many leaves did she measure?

2. Jennifer collected data on the length of time it took her to travel to school. She drew the histogram below.

    Time taken to travel to school
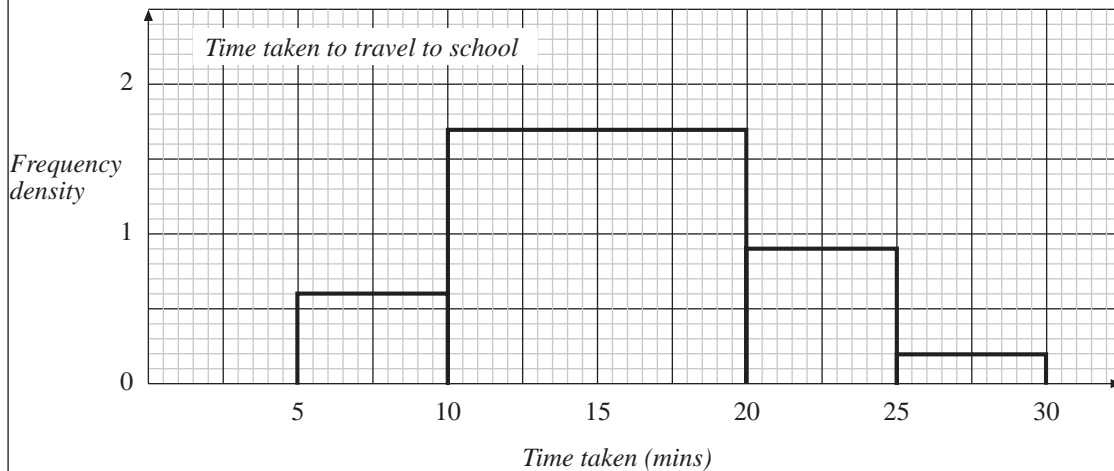
    This histogram contains an error. What is it?

3. Fred often travels by train. He kept a record of the time he had to wait when telephoning his local railway station to enquire about train times.

Waiting time (mins)	Frequency
$0 < t \le 0.5$	8
$0.5 < t \le 1.0$	10
$1.0 < t \le 3.0$	15
$3.0 < t \le 4.0$	12
$4.0 < t \le 5.0$	3

    Draw a histogram to show this data.

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4. A teacher recorded all the scores of the students who took a maths test in his school. These scores are summarised in the table below.

Score	Frequency
$0 \leq t < 35$	3
$35 \leq t < 40$	7
$40 \leq t < 55$	20
$55 \leq t < 70$	42
$70 \leq t \leq 100$	16

Draw a histogram for this data.

5. A survey was carried out to find the weekly income of a group of Year 11 pupils. The income includes pocket money and wages from part-time jobs.

Income (£)	Frequency
$0 < I \leq 10$	8
$10 < I \leq 15$	24
$15 < I \leq 20$	19
$20 < I \leq 40$	7

Draw a histogram to show this data.

6. The distribution of the ages of inhabitants of a village is shown in the table below.

Age	Frequency
0 - 4	10
5 - 9	12
10 - 19	19
20 - 39	36
40 - 59	30
60 - 64	9
65 - 79	11
80 - 99	3

(a) Explain why the width of the first class interval is 5.

(b) Find the width of all the other class intervals.

(c) Draw a histogram to show this data.

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7.   The finishing times to the nearest minute for the competitors in a half-marathon to complete the race are given below.

135	103	123	116	177	101
107	117	126	117	156	133
127	95	134	110	201	115
202	113	170	105	132	105
155	98	137	115	129	117
152	93	116	92	152	97
169	112	163	124	151	143
160	121	176	100	84	114
122	98	96	118	153	112
153	136	123	80	170	109

Group the data into suitable intervals and draw a histogram.

8.   A GCSE examiner recorded the time that it took to mark the essays students wrote in an exam. The times are in this table.

Time (mins)	Frequency
10 - 12	9
13 - 18	24
19 - 20	21
21 - 30	16
31 - 45	3

(a)   State the width of each class interval.

(b)   Draw a histogram to show this data.

9.   The age of each person in a coach party is illustrated in the histogram below.
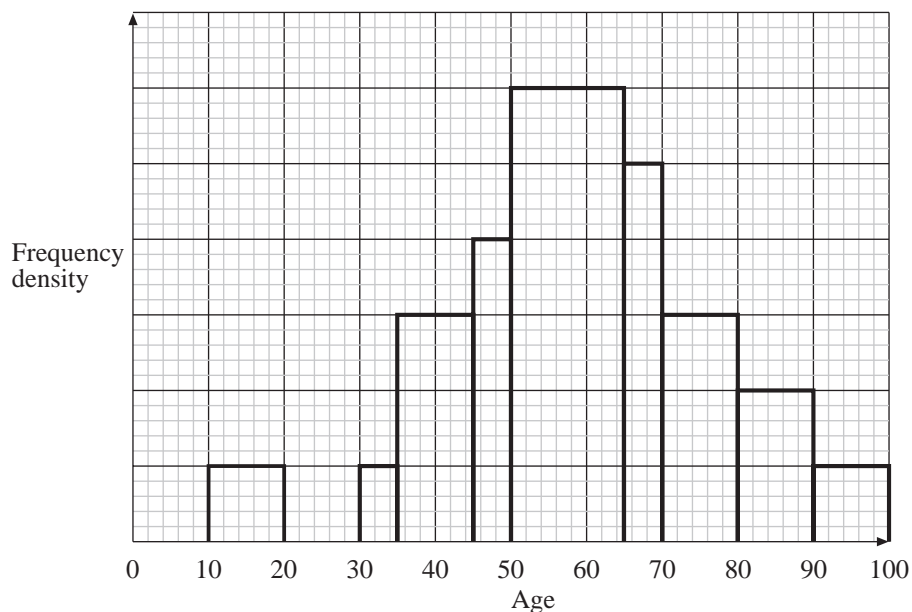
2.9

There are 6 people in the 70-80 age range.

(a)   How many people are there in the 45-50 age range?

(b)   How many people are there in the 50-70 age range?

(SEG)

10.   A sample was taken of the telephone calls to a school switchboard. The lengths of the telephone calls are recorded, in minutes, in this table.

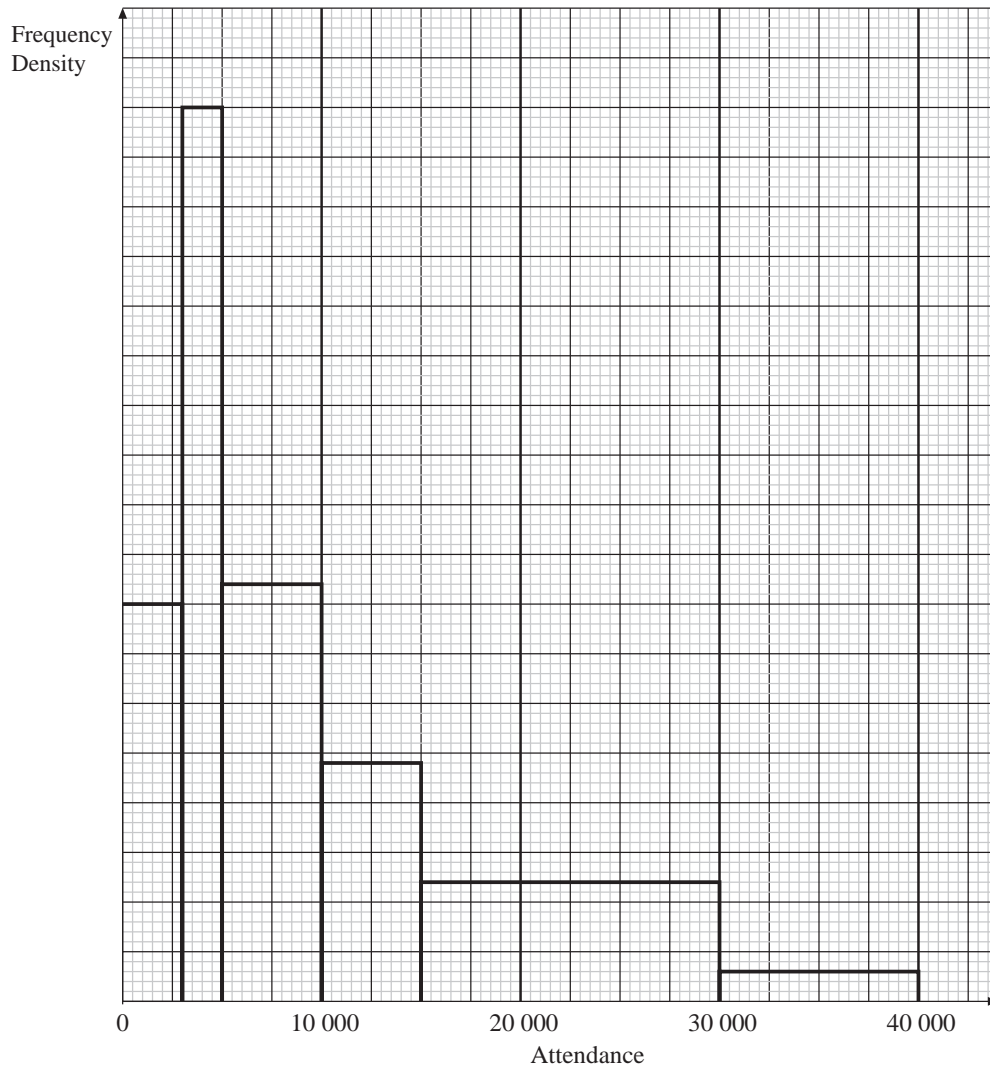| *Time in minutes (t)*	$0 < t \leq 1$	$1 < t \leq 3$	$3 < t \leq 5$	$5 < t \leq 10$	$10 < t \leq 20$
Number of calls	12	32	19	20	15

Copy and complete the histogram to show this information.
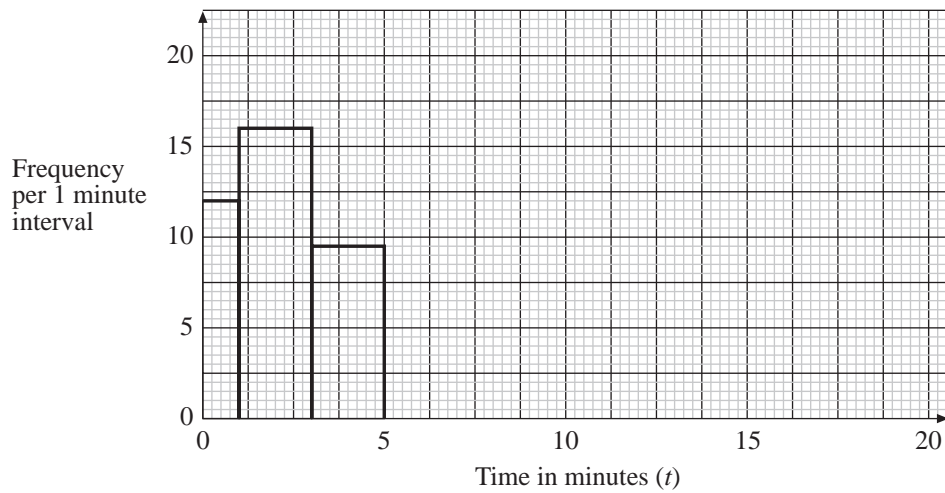
(MEG)

2.9

11.   The histogram below represents the number of spectators at professional football matches one Saturday.

No match had more than 40 000 spectators.

At 4 matches the number of spectators was greater than or equal to 10 000 and less than 15 000.

(a)   Use the information in the histogram to complete a copy of the frequency table below.

Number of spectators (n)	Frequency
$0 \leq n < 3000$	
$3000 \leq n < 5000$	
$5000 \leq n < 10\,000$	
$10\,000 \leq n < 15\,000$	4
$15\,000 \leq n < 30\,000$	
$30\,000 \leq n < 40\,000$	

(b)   Calculate the total number of professional football matches played in England on that Saturday.   *(LON)*

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