Name: ______________________________

GCSE (1 – 9)

Iteration

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 The number of rabbits in a field t days from now is P_t where

$$P_0 = 220$$
$$P_{t+1} = 1.15(P_t - 20)$$

Work out the number of rabbits in the garden 3 days from now.

$P_1 = 1.15(220 - 20) = 230$

$P_2 = 1.15(Ans - 20) = 242$

$P_3 = 1.15(Ans - 20) = 255$

nearest integer

255

(Total for question 1 is 3 marks)

2 The number of people living in a town t years from now is P_t where

$$P_0 = 55000$$
$$P_{t+1} = 1.03(P_t - 800)$$

Work out the number of people in the town 3 years from now.

$P_1 = 1.03(55000 - 800) = 55826$

$P_2 = 1.03(Ans - 800) = 56677$

$P_3 = 1.03(Ans - 800) = 57553$

(nearest integer)

57553

(Total for question 2 is 3 marks)

3 Using $x_{n+1} = 3 + \frac{9}{x_n^2}$

With $x_0 = 3$

Find the values of x_1, x_2 and x_3.

$$x_1 = 3 + \frac{9}{(3)^2} = 4$$

$$x_2 = 3 + \frac{9}{(Ans)^2} = 3.5625$$

$$x_3 = 3 + \frac{9}{(Ans)^2} = 3.709141274$$

$x_1 =$ 4

$x_2 =$ 3.5625

$x_3 =$ 3.709141274

(Total for question 3 is 3 marks)

4 Using $x_{n+1} = \frac{5}{x_n^2 + 3}$

With $x_0 = 1$

Find the values of x_1, x_2 and x_3.

$$x_1 = \frac{5}{(1)^2 + 3} = 1.25$$

$$x_2 = \frac{5}{(Ans)^2 + 3} = 1.095890411$$

$$x_3 = \frac{5}{(Ans)^2 + 3} = 1.190199669$$

$x_1 =$ 1.25

$x_2 =$ 1.095890411

$x_3 =$ 1.190199669

(Total for question 4 is 3 marks)

5 Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = \frac{7}{x_n^2} + 2$ three times to find an estimate for the solution to $x^3 - 2x^2 = 7$

$$x_1 = \frac{7}{(3)^2} + 2 = \frac{25}{9}$$

$$x_2 = \frac{7}{(Ans)^2} + 2 = 2.9072$$

$$x_3 = \frac{7}{(Ans)^2} + 2 = 2.82822478$$

2.82822478

(Total for question 5 is 3 marks)

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6 Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{2}{x_n^2 + 3}$ three times to find an estimate for the solution to $x^3 + 3x = 2$

$$x_1 = \frac{2}{(0)^2 + 3} = \frac{2}{3}$$

$$x_2 = \frac{2}{(Ans)^2 + 3} = \frac{18}{31}$$

$$x_3 = \frac{2}{(Ans)^2 + 3} = 0.5993140006$$

0.5993140006

(Total for question 6 is 3 marks)

7 Using $x_{n+1} = \frac{5}{x_n^2} + 2$

With $x_0 = 2.5$

(a) Find the values of x_1, x_2 and x_3.

$$x_1 = \frac{5}{(2.5)^2} + 2 = 2.8$$

$$x_2 = \frac{5}{(Ans)^2} + 2 = 2.637755102$$

$$x_3 = \frac{5}{(Ans)^2} + 2 = 2.718622914$$

$x_1 =$ 2.8

$x_2 =$ 2.637755102

$x_3 =$ 2.718622914

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 - 2x^2 - 5 = 0$

$$x^2(x-2) - 5 = 0$$

$$x^2(x-2) = 5$$

$$x - 2 = \frac{5}{x^2}$$

$$x = \frac{5}{x^2} + 2$$

$x = \frac{5}{x^2} + 2$ is a rearrangement of $x^3 - 2x^2 - 5 = 0$
x_1, x_2 and x_3 are estimates of ~~the~~ a solution to $x^3 - 2x^2 - 5 = 0$

(2)

(Total for question 7 is 5 marks)

8 (a) Show that the equation $2x^3 - x^2 - 3 = 0$ has a solution between $x = 1$ and $x = 2$.

when $x = 1$ $\quad 2(1)^3 - (1)^2 - 3 = -2$

when $x = 2$ $\quad 2(2)^3 - (2)^2 - 3 = 9$

one positive and one negative $\therefore$ solution between 1 and 2.

(2)

(b) Show that the equation $2x^3 - x^2 - 3 = 0$ can be rearranged to give: $x = \sqrt{\dfrac{3}{2x-1}}$

$$x^2(2x-1) - 3 = 0$$
$$x^2(2x-1) = 3$$
$$x^2 = \frac{3}{2x-1}$$
$$x = \sqrt{\frac{3}{2x-1}}$$

(1)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \sqrt{\dfrac{3}{2x_n - 1}}$ twice to find an estimate for the solution to $2x^3 - x^2 - 3 = 0$

$$x_1 = \sqrt{\frac{3}{2(1)-1}} = \sqrt{3}$$

$$x_2 = \sqrt{\frac{3}{2(\text{Ans})-1}} = 1.103395785$$

1.103395785

(3)

(Total for question 8 is 6 marks)

9 Using $x_{n+1} = 1 + \frac{1}{x_n^2}$

With $x_0 = 2$

(a) Find the values of x_1, x_2 and x_3.

$$x_1 = 1 + \frac{1}{(2)^2} = 1.25$$

$$x_2 = 1 + \frac{1}{(Ans)^2} = 1.64$$

$$x_3 = 1 + \frac{1}{(Ans)^2} = 1.371802499$$

x_1 = 1.25

x_2 = 1.64

x_3 = 1.371802499

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 - x^2 - 1 = 0$

$$x^2(x-1) - 1 = 0$$

$$x^2(x-1) = 1$$

$$x - 1 = \frac{1}{x^2}$$

$$x = 1 + \frac{1}{x^2}$$

$x = 1 + \frac{1}{x^2}$ is a rearrangement of $x^3 - x^2 - 1 = 0$

x_1, x_2 and x_3 are estimates of a solution to $x^3 - x^2 - 1 = 0$.

(2)

(Total for question 9 is 5 marks)

10 (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$.

$x^3 + 4x - 1 = 0$

when $x = 0$ $\quad (0)^3 + 4(0) - 1 = -1$

$x = 1$ $\quad (1)^3 + 4(1) - 1 = 4$

one positive, one negative ∴ solution between 0 and 1

(2)

(b) Show that the equation $x^3 + 4x = 1$ can be rearranged to give: $x = \frac{1}{4} - \frac{x^3}{4}$

$4x = 1 - x^3$

$x = \frac{1}{4} - \frac{x^3}{4}$

(1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice to find an estimate for the solution to $x^3 + 4x = 1$

$x_1 = \frac{1}{4} - \frac{(0)^3}{4} = 0.25$

$x_2 = \frac{1}{4} - \frac{(Ans)^3}{4} = 0.24609375$

0.24609375

(3)

(Total for question 10 is 6 marks)