Name: ______________________________

GCSE (1 – 9)

Proof of Circle Theorems

Instructions

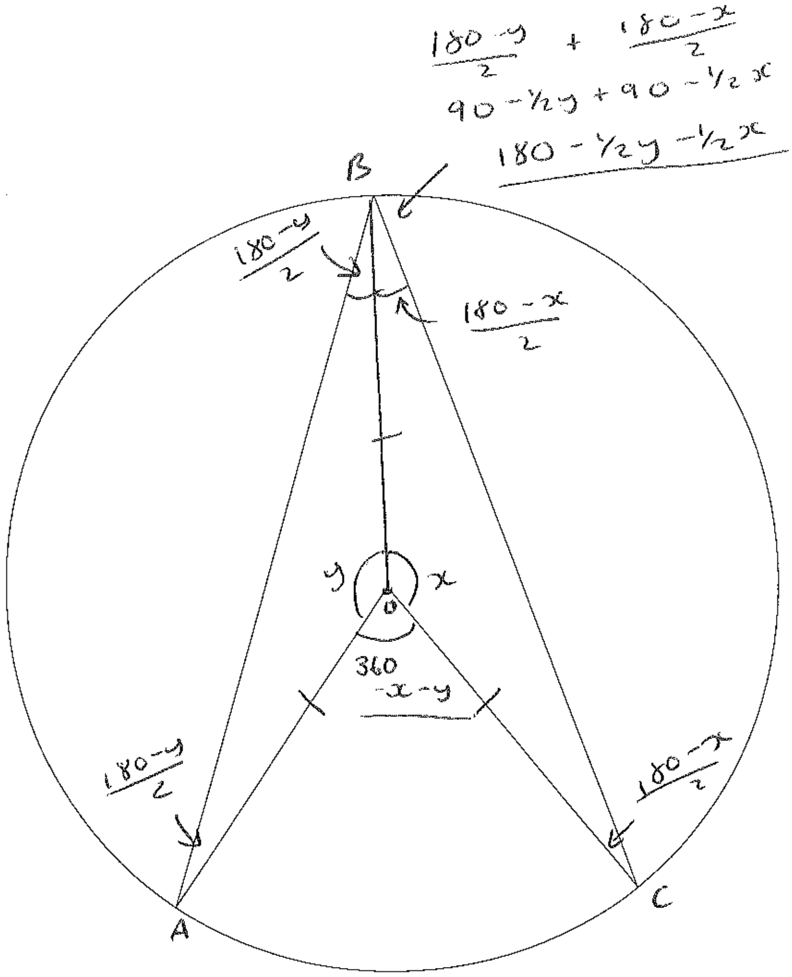
- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end



Prove that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference

Let $BOC = x$

$AOB = y$

$\therefore AOC = 360 - x - y$

Angles CBO and BCO $= \frac{180-x}{2}$ (angles in isosceles triangle)

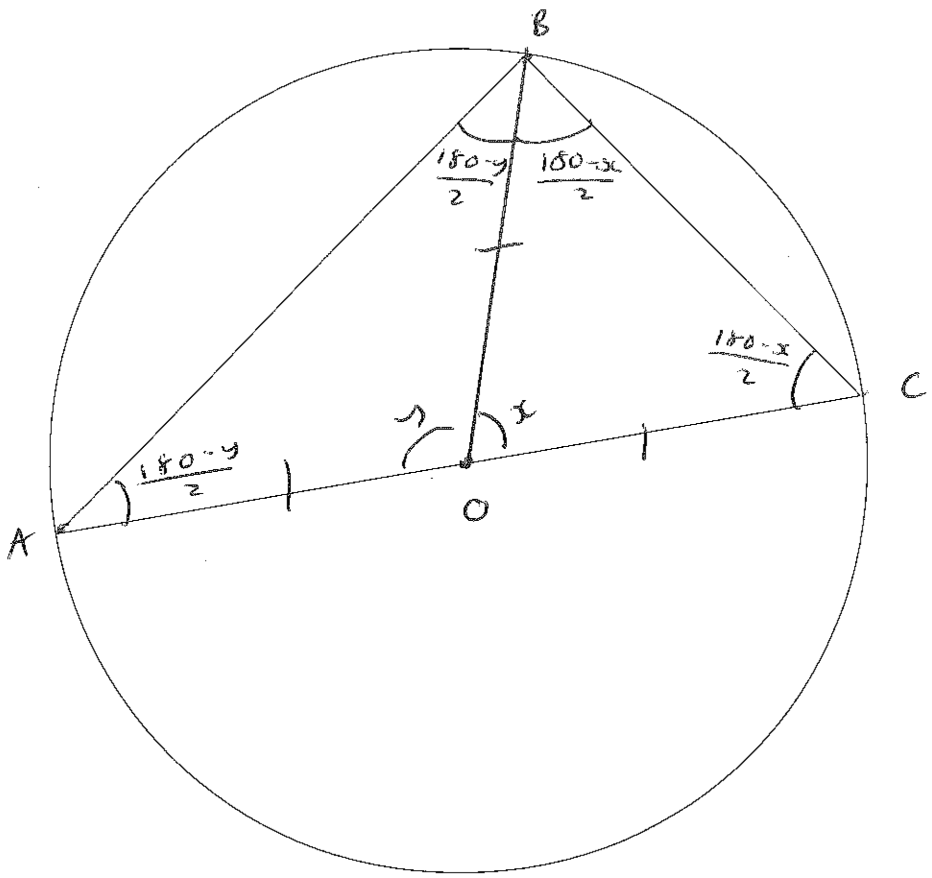
Angles BAO and $ABO = \frac{180-y}{2}$ — " —

Angle $ABC = \frac{180-y}{2} + \frac{180-x}{2}$

$90 - \frac{1}{2}y + 90 - \frac{1}{2}x$

$180 - \frac{1}{2}x - \frac{1}{2}y$

$360 - x - y = 2(180 - \frac{1}{2}x - \frac{1}{2}y)$ (4)



Prove the angle subtended at the circumference by a semicircle is a right angle

Let $AOB = y$ and $BOC = x$

$$x + y = 180^\circ$$

Angles ~~BO~~: ABO and $BAO = \frac{180-y}{2}$

Angles BCO and $CBO = \frac{180-x}{2}$

(Angles in isosceles triangle)

$$ABC = \frac{180-y}{2} + \frac{180-x}{2}$$

$$= 90 - \tfrac{1}{2}y + 90 - \tfrac{1}{2}x$$

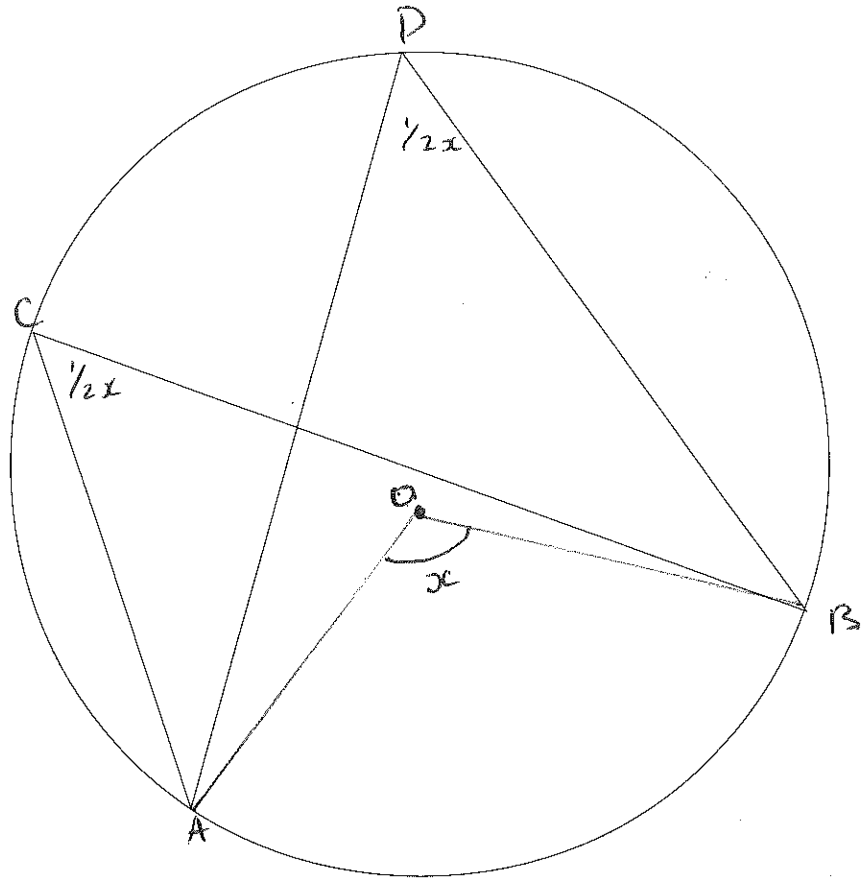
$$= 180 - \tfrac{1}{2}y - \tfrac{1}{2}x$$

(As $x + y = 180$ $\quad \tfrac{1}{2}x + \tfrac{1}{2}y = 90$)

$$= 180 - (\tfrac{1}{2}x + \tfrac{1}{2}y)$$

$$= 180 - 90 = 90^\circ$$

(4)



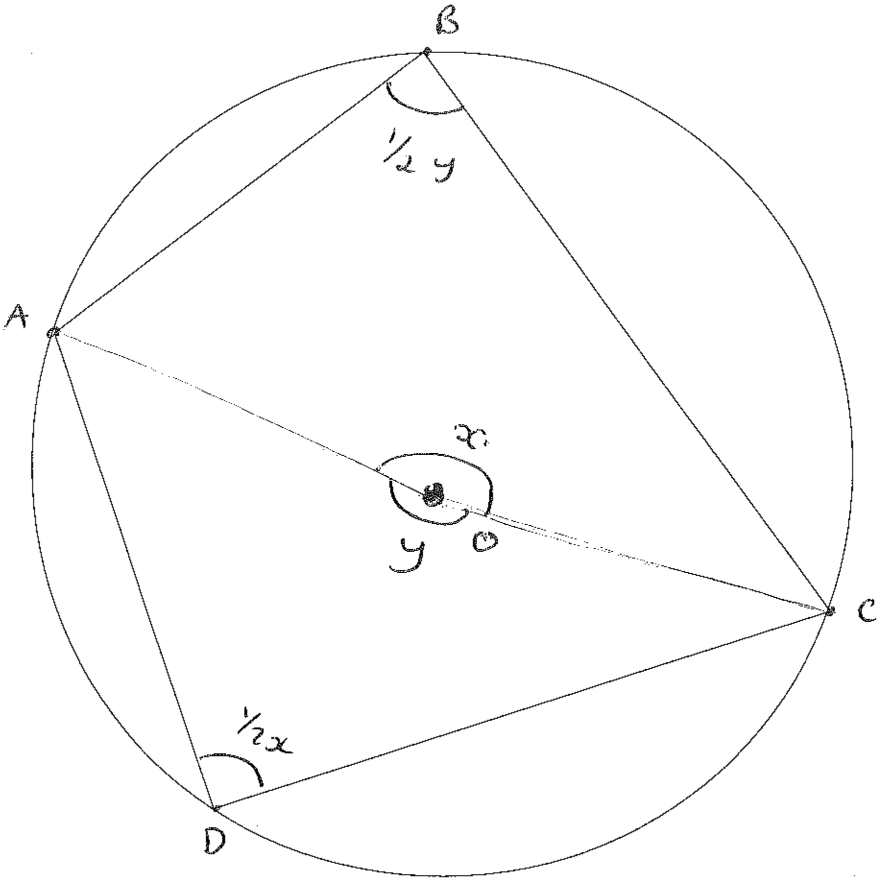
Prove that angles in the same segment are equal

Angle AOB $= x$

ACB and ADB $= \frac{1}{2}x$ (angles at circumference are half angles at centre)

$\underline{\frac{1}{2}x = \frac{1}{2}x}$

(4)



Prove that opposite angles of a cyclic quadrilateral sum to 180°

Let angle AOC (minor) $= x$

Let angle AOC (major) $= y$

Angle $x + y = 360°$ (angles at a point)

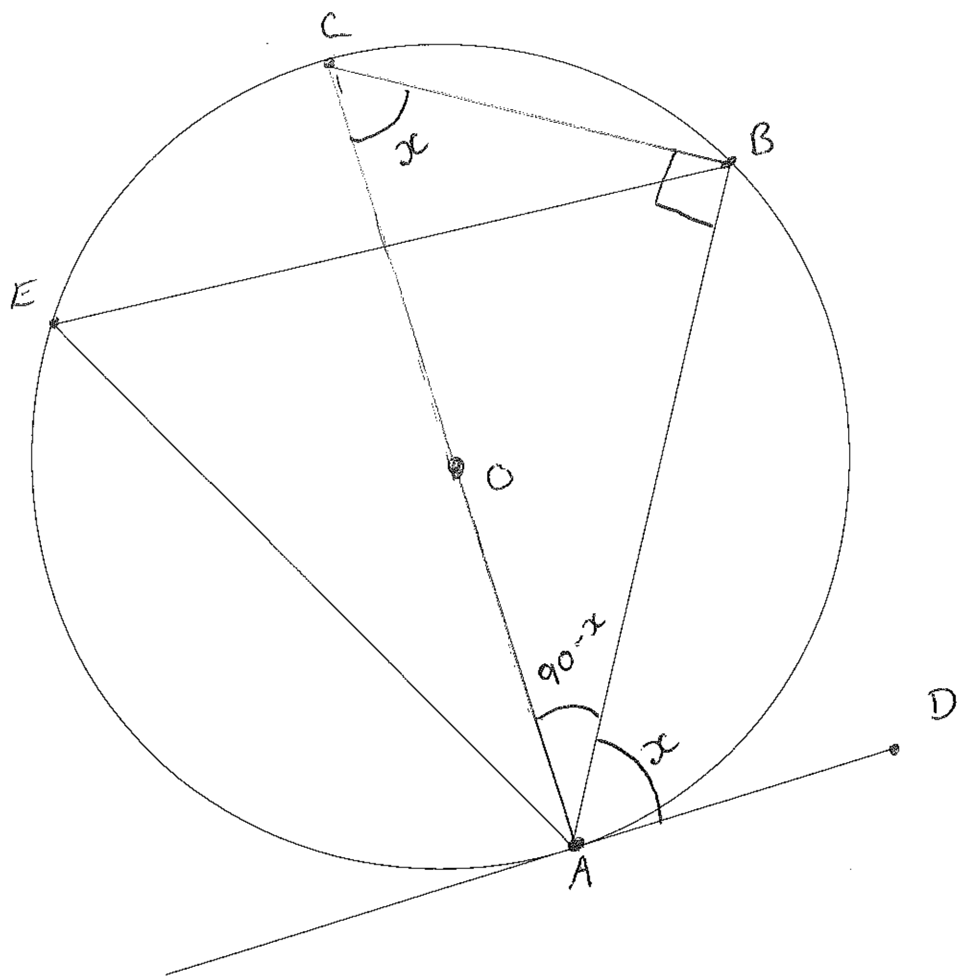
$A\hat{D}C = \frac{1}{2}x$ (Angle at circumference is half angle at centre.

$A\hat{B}C = \frac{1}{2}y$

As $x + y = 360$

$\frac{1}{2}x + \frac{1}{2}y = 180$

(4)



Prove the alternate segment theorem

The angle where tangent meets radius is 90°

The angle in a semi circle is 90°

Let angle BAD $= x$

$\therefore$ Angle BAC $= 90 - x$ (tangent meets radius)

$\therefore$ Angle ACB $= x$ (Angles in a triangle add up to 180° $180 - 90 - (90 - x) = x$)

Angle AEB also $= x$ (Angles in same segment are equal)

(4)