

# Edexcel GCSE

## Mathematics (Linear) – 1MA0

# CIRCLE THEOREMS

### Materials required for examination

Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser.  
Tracing paper may be used.

### Items included with question papers

Nil



### Instructions

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Use black ink or ball-point pen.

Fill in the boxes at the top of this page with your name, centre number and candidate number.

Answer all questions.

Answer the questions in the spaces provided – there may be more space than you need.

Calculators may be used.

### Information

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The marks for each question are shown in brackets – use this as a guide as to how much time to spend on **each** question.

Questions labelled with an **asterisk** (\*) are ones where the quality of your written communication will be assessed – you should take particular care on these questions with your spelling, punctuation and grammar, as well as the clarity of expression.

### Advice

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Read each question carefully before you start to answer it.

Keep an eye on the time.

Try to answer every question.

Check your answers if you have time at the end.

1.

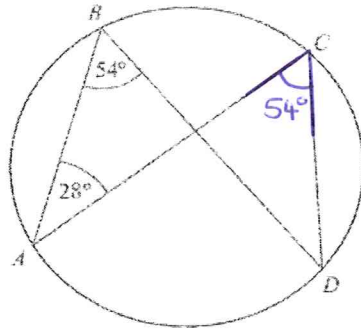


Diagram NOT accurately drawn

$A, B, C$  and  $D$  are points on the circumference of a circle.

Angle  $ABD = 54^\circ$ .

Angle  $BAC = 28^\circ$ .

(i) Find the size of angle  $ACD$ .

.....54.....<sup>o</sup>

(ii) Give a reason for your answer.

.....Angles in the same segment are equal.....

(3 marks)

2.

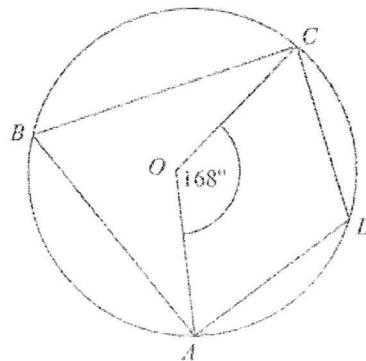


Diagram NOT accurately drawn

$A, B, C$  and  $D$  are points on the circumference of a circle, centre  $O$ .

Angle  $AOC = 168^\circ$

Work out the size of angle  $ADC$ .

You must give reasons for your working.

$\angle ABC = \frac{1}{2}(168) = 84^\circ$  Angles at the centre are twice angles at the circumference.

$\angle ADC = 180 - \angle ABC$   
 $= 180 - 84$  Opposite angles in a cyclic quadrilateral have a sum of  $180^\circ$ .  
 $= 96^\circ$

(4 marks)

3.

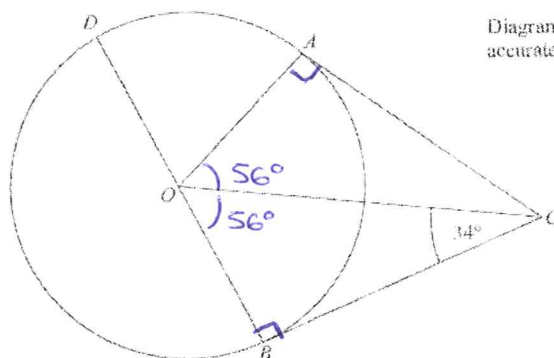


Diagram NOT accurately drawn

$A, B$  and  $D$  are points on the circumference of a circle, centre  $O$ .  
 $BOD$  is a diameter of the circle.  
 $BC$  and  $AC$  are tangents to the circle.  
 Angle  $OCB = 34^\circ$ .

Work out the size of angle  $DOA$ .

$\angle CBO = 90^\circ$  A radius and tangent meet at  $90^\circ$  on the circumference of a circle.

$\angle BOC = 180 - (34 + 90)$  Angles in a triangle have a sum of  $180^\circ$ .  
 $= 180 - 124$   
 $= 56^\circ$

$\angle BOC = \angle COA$  as  $\triangle BOC$  and  $\triangle AOC$  are congruent

$\angle DOA = 180 - (\angle BOC + \angle COA)$  Angles on a straight line have a sum of  $180^\circ$  ..... **68** .....  
 $= 180 - (56 + 56) = 68^\circ$  ..... **(4 marks)**

4.

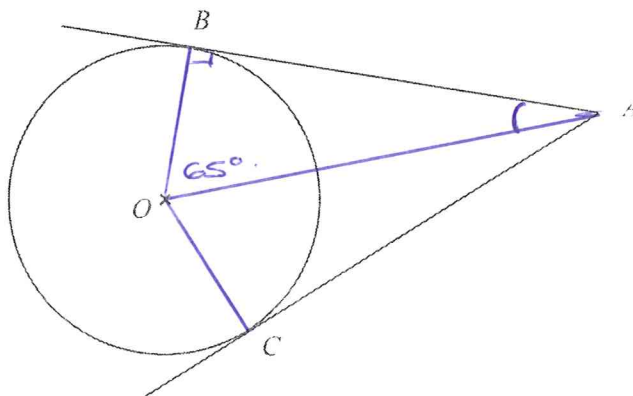


Diagram NOT accurately drawn

$B$  and  $C$  are points on a circle, centre  $O$ .  
 $AB$  and  $AC$  are tangents to the circle.  
 Angle  $BOC = 130^\circ$ .

Work out the size of angle  $BAO$ .

$\angle BOA = \frac{1}{2} \angle BOC = \frac{1}{2} (130) = 65^\circ$

$\angle OBA = 90^\circ$  Tangent and radius meeting at circumference

$\angle BAO = 180 - (65 + 90)$   
 $= 180 - 155$   
 $= 25^\circ$   
 .....  
**(4 marks)**

5.

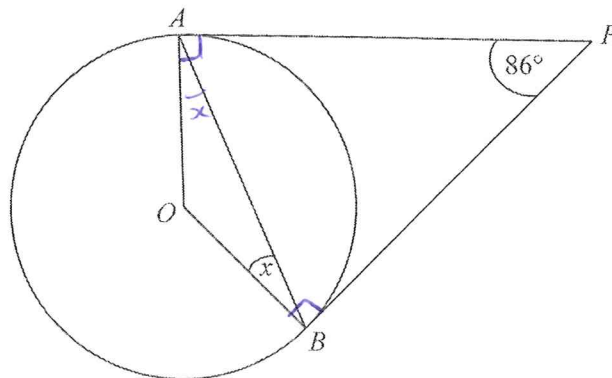


Diagram NOT accurately drawn

$A$  and  $B$  are points on the circumference of a circle, centre  $O$ .  
 $PA$  and  $PB$  are tangents to the circle.  
 Angle  $APB$  is  $86^\circ$ .

Work out the size of the angle marked  $x$ .

$$\begin{aligned} \angle PAO &= \angle PBO = 90^\circ \text{ (Tangent and radius meeting)} \\ \angle BOA &= 360 - (90 + 90 + 86) \\ &= 94^\circ \end{aligned}$$

$\triangle BOA$  is isosceles as  $OB = OA$  (both radii)

$$180 = 94 + 2x \Rightarrow 2x = 86 \\ x = 43^\circ$$

..... 43 .....

(3 marks)

6.

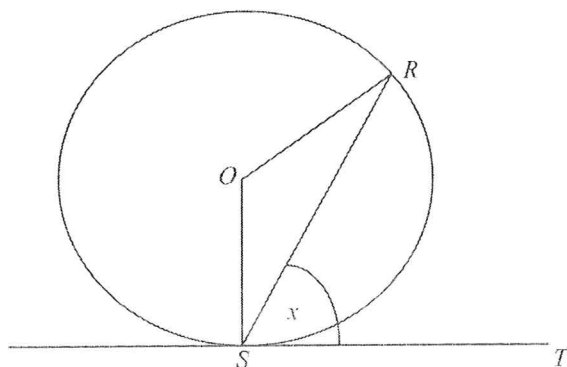


Diagram NOT accurately drawn

$R$  and  $S$  are two points on a circle, centre  $O$ .  
 $TS$  is a tangent to the circle.  
 Angle  $RST = x$ .

Prove that angle  $ROS = 2x$ .

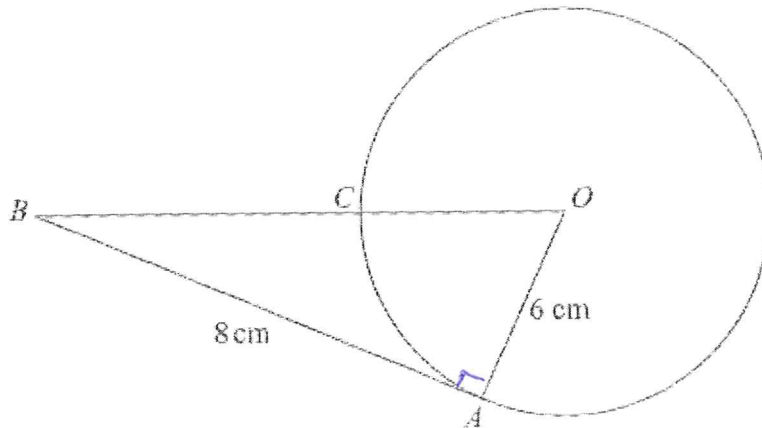
You must give reasons for each stage of your working.

$$\begin{aligned} \angle OST &= 90^\circ \text{ Tangent and Radius meeting at a circumference.} \\ \angle RSO &= 90 - x \\ \angle SRO &= \angle RSO = 90 - x \text{ as } \triangle RSO \text{ is isosceles, } RO = SO \text{ (both radii)} \\ \angle ROS &= 180 - (90 - x) - (90 - x) \text{ Sum of angles in a triangle is } 180^\circ \\ &= 180 - 90 + x - 90 + x \\ &= 2x \end{aligned}$$

(4 marks)

7.

Diagram NOT  
accurately drawn



In the diagram,  $O$  is the centre of the circle.  
 $A$  and  $C$  are points on the circumference of the circle.  
 $BCO$  is a straight line.  
 $BA$  is a tangent to the circle.

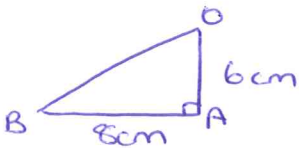
$AB = 8$  cm.  
 $OA = 6$  cm.

(a) Explain why angle  $OAB$  is a right angle.

A tangent and radius meet at  $90^\circ$  on the circumference of a  
 circle.

(1)

(b) Work out the length of  $BC$ .



Using Pythagoras

$$\begin{aligned} OB^2 &= 6^2 + 8^2 \\ &= 100 \\ OB &= \sqrt{100} \\ &= 10 \text{ cm} \end{aligned}$$

$OC = 6$  cm (radius of the circle).

$$\begin{aligned} \text{so } BC &= 10 - 6 \\ &= \underline{\underline{4}} \end{aligned}$$

..... 4 ..... cm

(3)

(4 marks)

8.

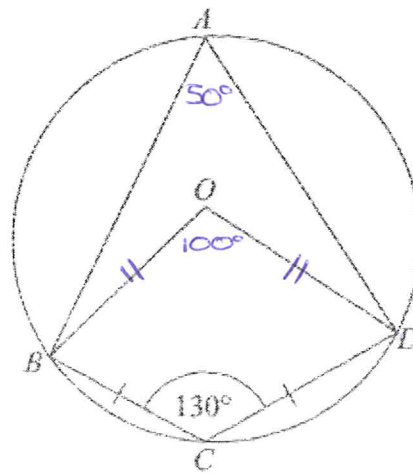


Diagram NOT accurately drawn

$A, B, C$  and  $D$  are points on a circle, centre  $O$ .  
 $BC = CD$ .  
 Angle  $BCD = 130^\circ$ .

- (a) Write down the size of angle  $BAD$ .  
 Give a reason for your answer.

$$\begin{aligned} \angle BAD &= 180 - 130^\circ \\ &= 50^\circ \end{aligned}$$

Opposite angles in a cyclic quadrilateral have a sum of  $180^\circ$ .

.....50.....<sup>o</sup>  
 (2)

- (b) Work out the size of angle  $ODC$ .  
 Give reasons for your answer.

$$\begin{aligned} \angle BOD &= 2 \times \angle BAD \\ &= 2 \times 50 \\ &= 100^\circ \end{aligned}$$

Angle at the centre twice that at the circumference.

$OBCD$  is a kite (pairs of adjacent equal sides).  
 therefore  $\angle OBC = \angle ODC$ . ( $= x$ )

so  $360 - 100 - 130 = 2x$  Angles in a quadrilateral have a sum of  $360^\circ$ .

$$\begin{aligned} 2x &= 130 \\ \underline{x} &= \underline{65^\circ} \end{aligned}$$

.....65.....<sup>o</sup>  
 (4)

(6 marks)

9.

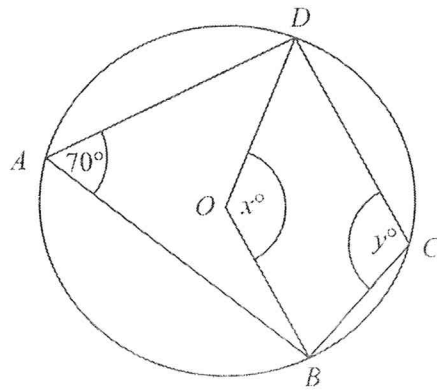


Diagram NOT accurately drawn

In the diagram,  $A, B, C$  and  $D$  are points on the circumference of a circle, centre  $O$ .  
Angle  $BAD = 70^\circ$ .  
Angle  $BOD = x^\circ$ .  
Angle  $BCD = y^\circ$ .

(a) (i) Work out the value of  $x$ .

$$x = \dots 140^\circ \dots$$

(ii) Give a reason for your answer.

Angle at the centre is twice the angle at the circumference

(2)

(b) (i) Work out the value of  $y$ .

$$y = \dots 110^\circ \dots$$

(ii) Give a reason for your answer.

Opposite angles in a cyclic quadrilateral have a sum of  $180^\circ$

(2)

(4 marks)

10.

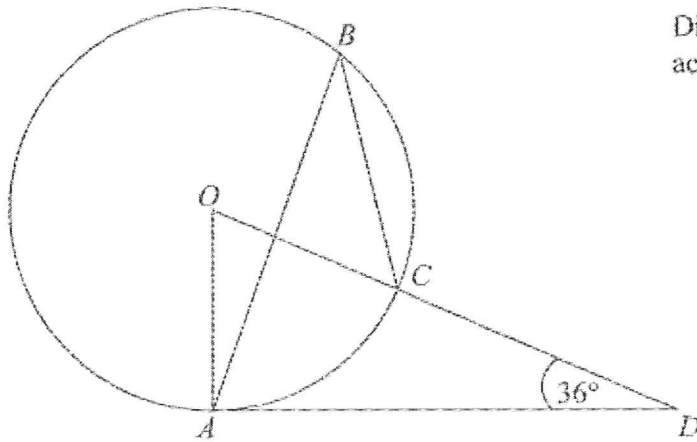


Diagram NOT accurately drawn

The diagram shows a circle centre  $O$ .  
 $A$ ,  $B$  and  $C$  are points on the circumference.

$DCO$  is a straight line.  
 $DA$  is a tangent to the circle.

Angle  $ADO = 36^\circ$

(a) Work out the size of angle  $AOD$ .

$$\angle DAO = 90^\circ \quad (\text{Radius and tangent meet at right angles at the circumference of a circle})$$

$$\angle AOD = 180 - 90 - 36 \quad (\text{Sum of angles in a triangle is } 180^\circ)$$

$$= 54^\circ$$

..... 54 .....  
 (2)

(b) (i) Work out the size of angle  $ABC$ .

$$\begin{aligned} \angle ABC &= \frac{1}{2} \angle AOC \\ &= \frac{1}{2} \angle AOD \\ &= \frac{1}{2} (54) \\ &= 27^\circ \end{aligned}$$

..... 27 .....  
 (3)

(ii) Give a reason for your answer.

..... The angle at the centre is twice the angle at the circumference, .....

..... So  $\angle AOC = 2 \angle ABC$  .....

.....

(3)  
 (4 marks)



11.

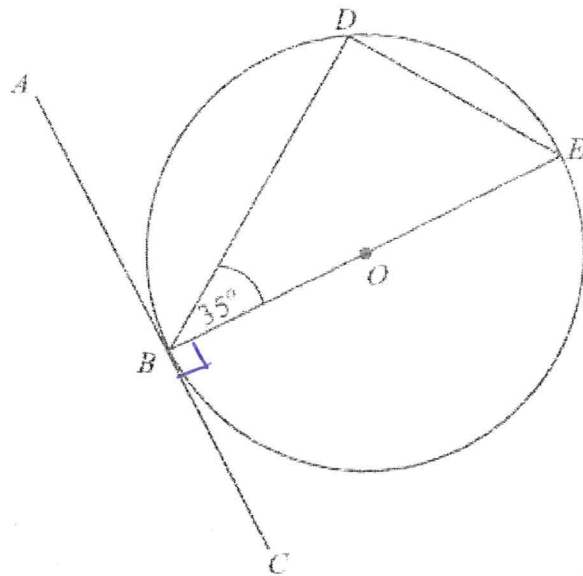


Diagram NOT accurately drawn

$B, D$  and  $E$  are points on a circle centre  $O$ .  
 $ABC$  is a tangent to the circle.  
 $BE$  is a diameter of the circle.  
 Angle  $DBE = 35^\circ$ .

(a) Find the size of angle  $ABD$ .

Give a reason for your answer.

$\angle CBE = 90^\circ$  Tangent and Radius meet at  $90^\circ$  at the circumference of a circle  
 $\angle ABD = 180 - (90 + 35)$  Angles on a straight line have a sum of  $180^\circ$   
 $= 180 - 125$   
 $= 55^\circ$

..... 55 .....  
 (2)

(b) Find the size of angle  $DEB$ .

Give a reason for your answer.

$\angle BDE = 90^\circ$  Angle opposite the diameter in a semi-circle.  
~~40~~  
 $\angle DEB = 180 - (90 + 35)$  Angles in a triangle have a sum of  $180^\circ$   
 $= 180 - 125$   
 $= 55^\circ$

..... 55 .....  
 (2)

(4 marks)

12.

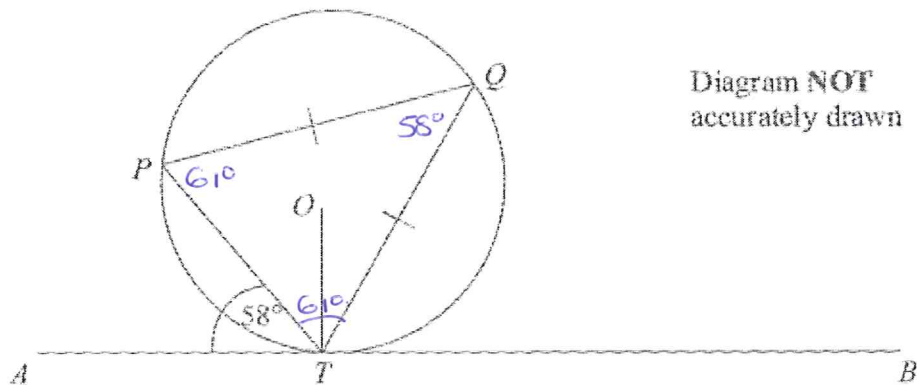


Diagram NOT accurately drawn

$P$ ,  $Q$  and  $T$  are points on the circumference of a circle, centre  $O$ .  
The line  $ATB$  is the tangent at  $T$  to the circle.

$PQ = TQ$ .  
Angle  $ATP = 58^\circ$ .

Calculate the size of angle  $OTQ$ .  
Give a reason for each stage in your working.

$$\angle TQR = 58^\circ \quad \text{Alternate Segment Theorem}$$

$$\angle QPT = \angle PTO \quad \text{Isosceles triangle}$$

$$\angle QPT = \frac{180 - 58}{2} = 61^\circ$$

$$\angle OTQ = 61 - \angle OTP$$

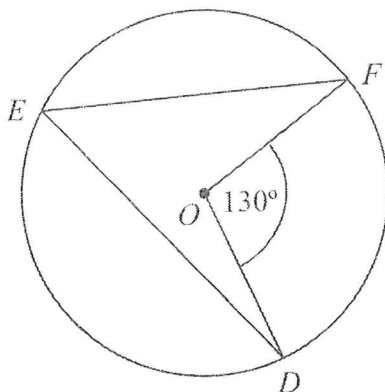
$$\begin{aligned} \angle OTP &= 90^\circ - 58^\circ \quad \text{Tangent and radius meet at } 90^\circ \\ &= 22^\circ \end{aligned}$$

$$\begin{aligned} \angle OTQ &= 61 - 22^\circ \\ &= 39^\circ \end{aligned}$$

.....39.....°

(4 marks)

13. (a)



$D, E$  and  $F$  are points on the circumference of a circle, centre  $O$ .  
Angle  $DOF = 130^\circ$ .

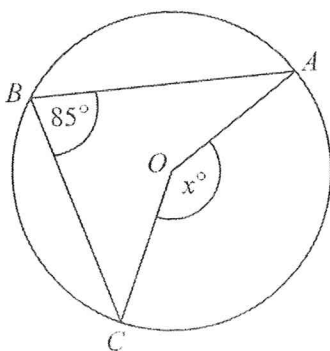
(i) Work out the size of angle  $DEF$ .

..... 65 ..... $^\circ$

(ii) Give a reason for your answer.

..... Angle at the circumference is half the angle at the  
..... centre .....

(2)



(b)

In the diagram,  $A, B$  and  $C$  are points on the circumference of a circle, centre  $O$ .  
Angle  $ABC = 85^\circ$ .

(i) Work out the size of the angle marked  $x^\circ$ .

..... 170 ..... $^\circ$

(ii) Give a reason for your answer.

..... Angle at the centre is twice the angle at the circumference .....

(2)

(4 marks)

\*14.

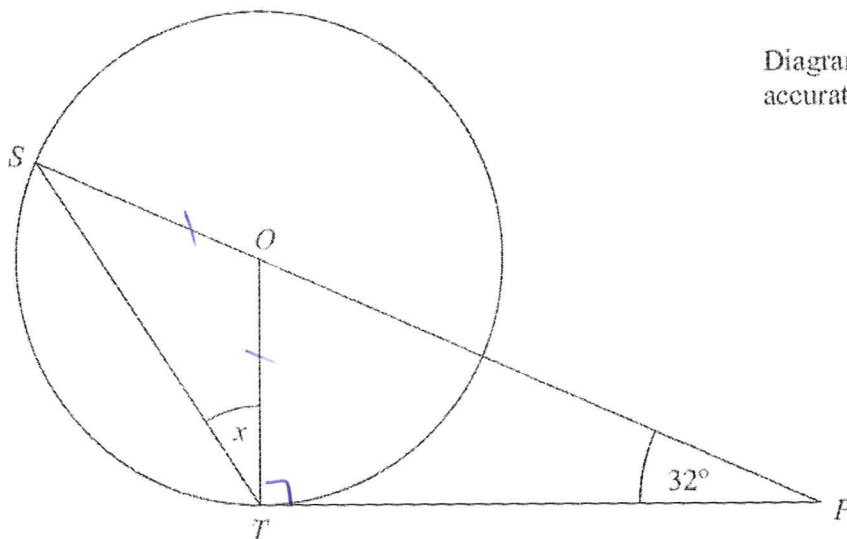


Diagram NOT  
accurately drawn

$S$  and  $T$  are points on the circumference of a circle, centre  $O$ .  
 $PT$  is a tangent to the circle.  
 $SOP$  is a straight line.

Angle  $OPT = 32^\circ$ .

Work out the size of the angle marked  $x$ .  
Give reasons for your answer.

$\angle PTO = 90^\circ$  Tangent and radius meet at  $90^\circ$  on  
the circumference of a circle.

$\angle POT = 180 - (90 + 32)$  Angles in a triangle have  
 $= 180 - 122$  a sum of  $180^\circ$ .  
 $= 58^\circ$ .

$\angle TOS = 180 - 58$  Angles on a straight line have  
 $= 122^\circ$  a sum of  $180^\circ$ .

$\triangle OTS$  is isosceles as  $OS = OT$  (both radii) so  $\angle OST = \angle STO$

$$\begin{aligned} 122 + 2x &= 180 \\ 2x &= 58 \\ x &= \underline{29^\circ} \end{aligned}$$

..... 29 .....

(Total 5 marks)