Name: ______________________________

GCSE (1 – 9)

Compound and Inverse Functions

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1. Given that $f(x)=x-4$ find:

a) $f(5)$

$f(5) = 5-4$

1 .... (1)

b) $f(3)$

$f(3) = 3-4$

-1 .... (1)

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2. Given that $g(x)=2x^2-10$ find:

a) $g(2)$

$g(2) = 2(2)^2 - 10$

$= 8 - 10$

-2 .... (1)

b) $g(-2)$

$g(-2) = 2(-2)^2 - 10$

$= 8 - 10$

-2 .... (1)

c) *Solve*: $g(x)=8$

$2x^2 - 10 = 8$

$2x^2 = 18$

$x^2 = 9$

$x = \pm 3$ .... (3)

3. Given that $f(x)=3x-5$ find:

a) $f(3)$ $= 3(3)-5$

$= 9-5$

4 (1)

b) $f(-2)$ $= 3(-2)-5$

$= -6-5$

-11 (1)

c) *Solve*: $f(x)=1$

$3x-5=1$

$3x=6$

$x=2$

.... (2)

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4. Given that $f(x)=x^2-3$ find:

a) $f(10)$ $= (10)^2-3$

$= 100-3$

97 (1)

b) $f(-1)$ $= (-1)^2-3$

$= 1-3$

-2 (1)

c) *Find*: $f^{-1}(x)$

$y = x^2-3$

$y+3 = x^2$

$\sqrt{y+3} = x$

$f^{-1}(x) = \sqrt{x+3}$

$f^{-1}(x) = \sqrt{x+3}$ (2)

5. Given that $f(x)=2x-4$ and $g(x)=3x+5$

a) Find: $gf(3)$

$f(3) = 2(3) - 4$
$= 6 - 4$
$= 2$

$g(2) = 3(2) + 5$
$= 6 + 5$

..... 11 ..... (2)

b) Work out an expression for: $f^{-1}(x)$

$y = 2x - 4$
$y + 4 = 2x$
$\frac{1}{2}(y+4) = x$
$f^{-1}(x) = \frac{1}{2}(x+4)$

$f^{-1}(x) = \frac{1}{2}(x+4)$ (2)

c) Solve: $f(x)=g(x)$

$2x - 4 = 3x + 5$
$-4 = x + 5$
$x = -9$

$x = -9$ ..... (2)

6. Given that $f(x)=3x+1$ and $g(x)=x^2$

a) Write down an expression for: $fg(x)$

$3x^2 + 1$ (2)

b) Work out an expression for: $gf(x)$

$(3x+1)^2$ (2)

c) Solve: $fg(x)=gf(x)$

$3x^2+1 = (3x+1)^2$

$3x^2+1 = 9x^2 + 3x + 3x + 1$

$0 = 6x^2 + 6x$

$0 = x^2 + x$

$0 = x(x+1)$

$x=0 \quad x=-1$

$x=0 \quad x=-1$ (3)

7. Given that $f(x)=x^2-17$ and $g(x)=x+3$

a) Work out an expression for: $g^{-1}(x)$

$y = x + 3$

$y - 3 = x$

$g^{-1}(x) = x - 3$ (2)

b) Work out an expression for: $f^{-1}(x)$

$y = x^2 - 17$

$y + 17 = x^2$

$\sqrt{y+17} = x$

$f^{-1}(x) = \sqrt{x+17}$ (2)

c) Solve: $f^{-1}(x)=g^{-1}(x)$

$\sqrt{x+17} = x - 3$

$x + 17 = (x-3)^2$

$x + 17 = x^2 - 6x + 9$

$0 = x^2 - 7x - 8$

$0 = (x-8)(x+1)$

$x = 8 \quad x = -1$

.... (4)

8. A function f is defined such that

$$f(x)=x^2-1$$

a) Find and expression for : $f(x-2)$

$f(x-2) = (x-2)^2 - 1$

$= x^2 - 2x - 2x + 4 - 1$

$= x^2 - 4x + 3$

x^2-4x+3 (2)

b) Hence solve: $f(x-2)=0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

$x = 3 \quad x = 1$ (2)

9. A function f is defined such that

$$f(x) = 4x - 1$$

a) Find: $f^{-1}(x)$

$$y = 4x - 1$$
$$y + 1 = 4x$$
$$\frac{y+1}{4} = x$$
$$f^{-1}(x) = \frac{x+1}{4}$$

…………… (2)

The function g is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant}$$

Given that $fg(2) = 12$

b) Work out the value of k

$$g(2) = k(2)^2$$
$$= 4k$$
$$f(4k) = 4(4k) - 1$$
$$= 16k - 1$$
$$16k - 1 = 12$$
$$16k = 13$$
$$k = \frac{13}{16}$$

…………… (2)